JOURNAL OF OPTOELECTRONICS AND ADVANCED MATERIALS Vol. 12, No. 1, January 2010, p. 6 - 11

Spatiotemporal vortex solitons in waveguide arrays

H. LEBLOND*, B. A. MALOMED[a], D. MIHALACHE[b]
Laboratoire de Photonique d'Angers, Université d'Angers, 2 Bd Lavoisier, 49000 Angers, France
[a]*Department of Physical Electronics, School of Electrical Engineering, Faculty of Electrical Engineering, Tel Aviv University, Tel Aviv 69978, Israel*
[b]*Horia Hulubei National Institute for Physics and Nuclear Engineering (IFIN-HH), Department of Theoretical Physics, 407 Atomistilor, Magurele-Bucharest 077125, Romania*

Two-dimensional arrays of optical waveguides, which feature dispersion, nonlinearity and discrete diffraction, support the propagation of spatiotemporal solitonic structures. Several structures with intrinsic vorticity S = 1 and 2 were studied by means of the variational analysis and numerical methods. Some vortices with S = 1 are stable. Numerical studies of interactions between them make it possible to identify four typical outcomes of the collisions, depending on the initial relative velocity of colliding solitons: rebound of slow solitons, fusion, splitting, and quasi-elastic interactions of fast solitons.

(Received December 15, 2009; accepted January 20, 2010)

Keywords: Spatiotemporal optical solitons, Vortex solitons, Waveguide arrays

1. Introduction

Theoretical and experimental studies of discrete spatial solitons (alias lattice solitons) have attracted a great deal of interest in the past years in the fields of optic and matter waves, see, e.g., reviews [1]-[4].

A fundamental model of lattice media amounts to the discrete nonlinear Schrödinger (DNLS) equation [5]. A direct realization of the one-dimensional (1D) DNLS model with the cubic nonlinearity in arrays of optical waveguides was first proposed in Ref. [6]. Later, the same DNLS equation was demonstrated to describe, in various theoretical and experimental settings, Bose-Einstein condensates (BECs) trapped in deep optical-lattice potentials, see, e.g., Ref. [7] for a review. Lattice vortex solitons, i.e., localized lattice excitations with embedded vorticity, that were predicted in Ref. [8], were created in 2D virtual photonic lattices [9]-[10]. In addition to the fundamental discrete vortices with topological charges $S=+1$ and $S=-1$, higher-order solitary vortices, with $|S|>1$, and multipole solitons, such as quadrupoles, were predicted in Refs. [11,12]. Two-dimensional (2D) spatial solitons were recently created [13] in a bundle of fiberlike waveguides permanently written in bulk silica by means of the technique based on tightly focused femtosecond laser pulses. Similarly structured quasidiscrete 2D solitons, including solitary vortices, were predicted in photonic crystal fibers [14].

It is worthy to mention that the settings based on fiber bundles, or arrays of waveguides written in bulk silica, feature a very fast response to variations of the light beams, which suggests a possibility to study spatiotemporal optical solitons (alias light bullets) in such physical settings, i.e., the temporal evolution in the longitudinal direction and quasidiscrete spatial patterns in the transverse direction; for a review of the topic of spatiotemporal solitons in nonlinear optics and BEC see Ref. [15]. Thus semidiscrete "light bullets" in arrays or fibers [16] and in photonic wires [17], self-compression [18] and steering [19] of pulsed beams, and the modulational instability [20] have been considered. Moreover, spatiotemporal optical solitons in models of waveguide arrays with the quadratic nonlinearity were studied too [21]. Semidiscrete spatiotemporal surface solitons were recently introduced in semi-infinite models of waveguide arrays [22] and at interfaces between two different waveguide arrays [23]. Available experimental techniques allow the creation of solitary modes in bundled arrays of optical fibers, thus the study of semidiscrete three-dimensional (3D) spatiotemporal optical solitons, as localized states which are continuous along the propagation axis and discrete in the transverse plane, is of much relevance. The same localized structures should be relevant to the description of 3D matter-wave solitons in a self-attractive BEC trapped in a deep 2D optical lattice [24]. Recently [25], we have reported results of a comprehensive analysis of spatiotemporal vortex solitons and quadrupoles in the semidiscrete 3D model. It was demonstrated that solitary vortices with "spin" (vorticity) $S=1$ and quadrupoles, in the form of rhombuses which are based on a set of four guiding cores, with an almost empty one at the center (*on-center vortices*), have a vast stability region. Solitary vortices of the "square" type, without an empty site in the middle (*off-center vortices*), feature a much smaller stability region, and all vortices with vorticity $S=2$ are unstable [25]. It is worthy to mention that a related work was performed [26] for vortex solitons with topological charges $S=1$ and $S=2$ in the continuum counterpart of the model, which includes a 2D periodic potential in the transverse plane. Families of discrete vortex solitons in the full DNLS equation in three dimensions were also reported [27]. Recently, we reported results of a systematic numerical analysis of collisions between localized 3D semidiscrete soliton complexes, viz., rhombus-shaped vortices, quadrupoles, and fundamental solitons, in the model of a bundle of fiberlike waveguides [28]. The model also describes a 3D self-attractive BEC loaded into a deep 2D optical lattice. Four generic outcomes of the collisions were identified: rebound of slow solitons, fusion, splitting, and also quasielastic interactions of fast solitons [28].

The present paper is organized as follows. In Sec. 2 we briefly overview the studies of existence and stability of 3D vortex solitons in optical fiber bundles. We consider complex spatiotemporal semidiscrete solitons in a model of a set of nonlinear optical fibers which form a square lattice in the cross section. The problem of collisions between localized 3D semidiscrete complexes, viz., rhombus-shaped vortices, quadrupoles, and fundamental solitons, in the model of a bundle of fiberlike waveguides is briefly reviewed in Sec. 3. The paper is concluded by Sec. 4.

2. Spatiotemporal vortex solitons in optical fibers bundles

In a recent work [25] we analyzed complex spatiotemporal semidiscrete solitons in a model of a set of nonlinear optical fibers which form a square lattice in the cross section. The nonlinear optical medium was recently realized as a set of parallel waveguides written in fused silica, see, e.g., Ref. [13]. As mentioned above, the model investigated in Ref. [25] may also apply to a self-attracting Bose-Einstein condensate trapped in a very strong quasi-2D optical lattice. By means of the variational approximation (VA) and using numerical methods, we constructed several species of the semidiscrete solitons, including vortices of the rhombus (alias cross, or on-site) and square (off-site) types, with vorticities $S=1$ and $S=2$, and quadrupoles. The VA was developed for narrow cross vortices with $S=1$ and quadrupoles, which turn out to be the two most stable species. Two finite stability intervals were also found for the square-shaped vortices with $S=1$, while all the vortices with $S=2$ were found to be unstable.

Generally, a vortex is represented by a pivotal point in the wavefield, around which there is a continuous circulation of a certain physical variable. In optics, the localized optical vortices (alias optical vortex solitons), have drawn much attention as objects of fundamental interest, and also due to their potential applications to all-optical information processing, as well as to the guiding and trapping of atoms. At the pivot of an optical vortex, the complex amplitude of the electromagnetic field vanishes, while circulation C of the phase of the complex field around an arbitrary closed contour surrounding the vortex core is a multiple of 2π, that is, $C=2\pi S$, where integer S is the same topological charge of the vortex ("spin") that was introduced above. Thus the phase dislocations carried by the wavefronts of light beams are associated with the zero-intensity point at the pivot; the wave is twisted around such points, creating an optical vortex.

Using adequate numerical methods, we have solved the coupled system of equations for the local amplitudes of the electromagnetic waves $u_{m,n}$ (z,τ) in the bundle of fiber waveguides [25] with the square-grid cross section, where the continuous variable τ is the reduced time, z is the propagation coordinate and (m,n) are discrete transverse coordinates in the waveguide array. We have found families of stationary solutions of the governing propagation equations [25] which are parametrized by propagation constant μ, looking for them as $u_{m,n}(z,\tau) = \exp(i\mu z)\, U_{m,n}(\tau)$, with functions $U_{m,n}(\tau)$ obeying a system of coupled ordinary differential equations. Recall that the definition of the vortex requires the phase of complex field $U_{m,n}(\tau)$ to change by $2\pi S$, with $S=1,2,3,\ldots$, as a result of a round trip around the center of the vortex. In this way, we have found two types of semidiscrete vortex solitons, *viz*., the above-mentioned on-site-centered "rhombuses" ("crosses") and off-site-centered "squares". The former species, with $S=1$, is based on the frame ("skeleton") composed of four lattice sites, with coordinates (m,n) = (1,0), (0,1), (-1,0), (0,-1), while the central site, at (m,n) = (0,0), remains empty. The "square"-shaped vortices do not include an empty site at the center, placing the "virtual" pivot of the vortex between lattice sites. Accordingly, the frame for the square vortex with $S=1$ is composed of four sites with coordinates (m,n) = (0,0), (1,0), (1,1), (0,1).

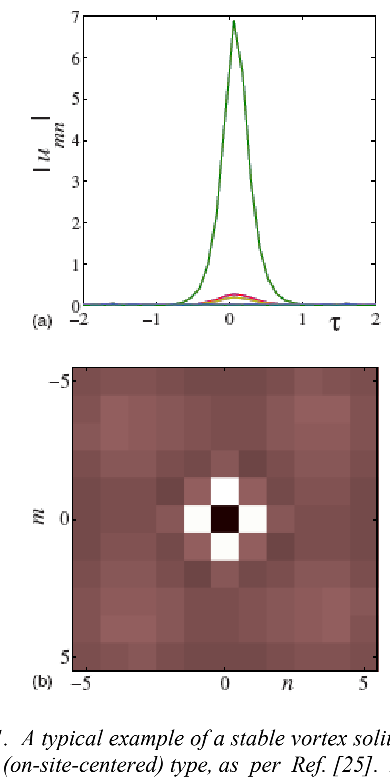

Fig. 1. A typical example of a stable vortex soliton of the cross (on-site-centered) type, as per Ref. [25]. Here the propagation constant is μ = 20.

A typical example of the stable vortex found slightly above the stability threshold [25], *viz*., at the value of the propagation constant μ = 20, is displayed in Fig. 1. It is worthy to notice that stable vortex solitons of this type are generated with propagation constant $\mu > \mu_{cr} = 19$. This figure includes a set of temporal profiles of the soliton in the continuous coordinate, $|U_{m,n}(\tau)|$, and a contour plot which shows the transverse distribution (on the square grid) of the single-site energy, $E = \int_{-\infty}^{+\infty} \left| U_{m,n}(\tau) \right|^2 d\tau$.

A typical example of a stable square-shaped vortex is shown in Fig. 2 for propagation constant μ = 5 (stable vortex solitons of this type were found in the region of $3 < \mu < 8$ [25]).

We have also investigated the stability of vortices with $S = 2$ and of quadrupoles. Although the quadrupole solitons carry no vorticity, they are akin to the vortices with $S = 2$. We have found that, similar to the situation of the case of 2D lattice solitons (without the longitudinal direction), the quadrupoles may be stable, while all vortices with $S = 2$, of either type (rhombuses or squares), are completely unstable [25]. A typical example of stable quadrupoles is displayed in Fig. 3 for propagation constant μ = 25 (the quadrupoles were found to be stable for μ > 20 [25]). As concerns unstable soliton species, various scenarios of the instability development were identified, that include the straightforward decay, merger of the complex semidiscrete localized pattern into a single fundamental soliton, or splitting into several mutually incoherent fundamental solitons, each carried, essentially, by a single core of the array [25]. In some cases, the eventual state may include stable coherent pairs of in-phase solitons (the latter outcome is characteristic to the instability of quadrupoles).

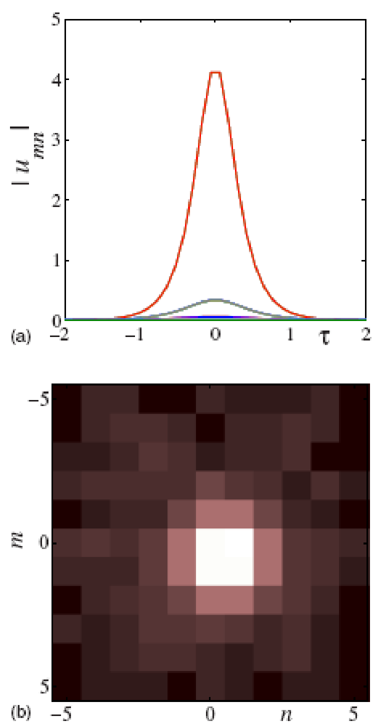

Fig. 2. A typical example of a stable vortex soliton of the square (off-site-centered) type, as per Ref. [25]. Here the propagation constant is μ = 5.

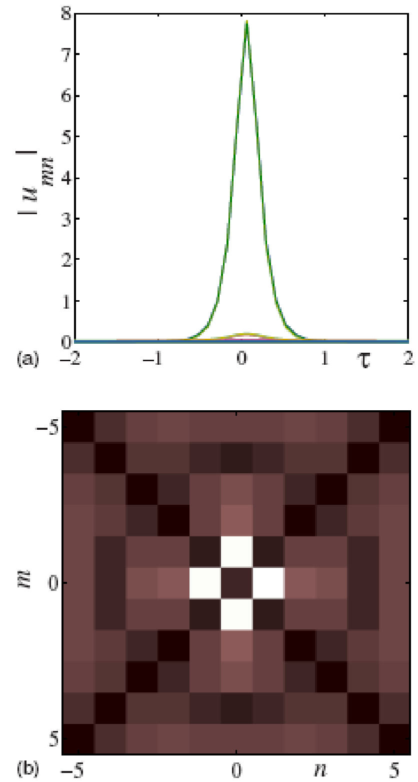

Fig. 3. A typical example of a stable quadrupole soliton. Here the propagation constant ia μ = 25, as per Ref.[25].

3. Interactions of spatiotemporal solitons and vortices in fiber bundles

Once stable soliton complexes with the topological structure have been found, an issue of straightforward interest is to consider collisions between them. In Ref. [28], we have recently performed a systematic analysis of collisions between solitons of the most robust types, i.e., rhombic vortices and quadrupoles, including collisions between the vortices with topological charges $(S_1, S_2) = (+1,+1)$ and $(+1,-1)$, that is, "co-rotating" and "counter-rotating" vortex pairs, as well as collisions between vortices and quadrupoles. In fact, the investigation of interactions of spatiotemporal solitons and vortices in fiber bundles is a generic example of the study of a 3D conservative model which makes it possible to consider collisions between vortex solitons in three dimensions. We also considered collisions between a rhombus-shaped vortex and fundamental soliton carried by the central waveguide of the bundle, which is nearly empty in the vortex state [28]. It is worthy to mention that very recently, collisions between coaxial 3D solitons with embedded vorticities were studied in the framework of the continual complex Ginzburg-Landau equation with the cubic-quintic nonlinearity for both co-rotating [29] and counter-rotating [30] configurations. Also, the generic collision scenarios between nonspinning and spinning coaxial 3D dissipative light bullets, described by the complex Ginzburg-Landau equation with the cubic-quintic nonlinearity, were recently presented [31].

It is worthy to mention that we recently analyzed the interactions between discrete surface light bullets in both 1D and 2D photonic lattices, where we observed a variety of collision scenarios and different outcomes, such as the soliton fusion, soliton switching, symmetric and asymmetric scattering [32]. Moreover, we recently considered continuous-discrete spatiotemporal models described by the complex Ginzburg-Landau equation [33]-[34]. Also, the domains of existence and stability of in-phase (unstaggered) on-site (single-peaked), inter-site (double-peaked) and flat-top-like (four-peaked) dissipative light bullets in 2D photonic lattices were determined, and various instability-induced scenarios of the dynamics of these discrete Ginzburg-Landau spatiotemporal optical solitons were described [35]. Further, systematic results of collisions between both discrete and continuous spatiotemporal Ginzburg-Landau solitons were reported in Refs. [36]-[37].

In the following we briefly present the generic results for collisions between "co-rotating" rhombus-shaped vortices, i.e., ones with equal topological charges, $S_1 = S_2 = 1$ [28]. Collisions between vortices with a large relative velocity naturally lead to their passage through each other. The passage is quasielastic, but not always completely elastic. From the typical example displayed in Fig. 4, we see that the collision gives rise to a slowly developing splitting of each rhombus into two pairs of fundamental solitons located at opposite vertices of the rhombus; in this case, only trajectories of the fundamental soliton components which build the vortices are shown in Fig. 4. At smaller velocities or larger values of the energy of the vortices, the collision becomes strongly inelastic, leading to a merger (fusion) of the vortices into a single one of the same type as shown in Fig. 5. At essentially smaller values of the relative velocity, repulsion between slowly moving vortices actually prevents the collision, and as a result, both vortices come to a halt, keeping a large temporal distance between them, see Fig. 6. In Ref. [28], we have also investigated collisions between "counter-rotating" rhombus-shaped vortices, i.e., ones with opposite topological charges, $S_1 = +1$ and $S_2 = -1$. In that case, the results are quite similar to those reported above for $S_1 = S_2 = 1$.

Collisions between identical quadrupoles give rise to outcomes of the same four types as identified above for rhombic vortices: (a) rebound of slowly moving objects, (b) fusion (full or partial fusion, in case of the collision between vortices with equal or opposite charges, respectively), (c) splitting (possibly with the formation of bound states of two solitons), and (d) quasielastic passage of fast solitons through each other [28].

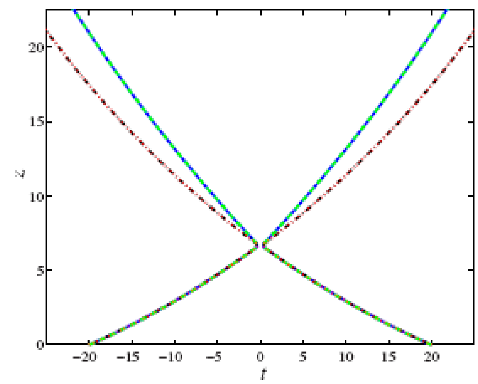

Fig. 4. A typical example of the quasielastic collision of two identical rhombus-shaped vortices, with topological charges $S_1 = S_2 = 1$, propagation constants $\mu_1 = \mu_2 = 20$, and relative velocity $\Delta V = 8$, as per Ref. [28].

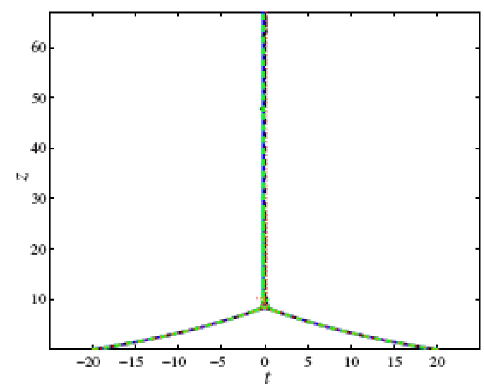

Fig. 5. A typical example of the merger (fusion) resulting from the collision of identical rhombic vortices, with propagation constants $\mu = 22$ and relative velocity $\Delta V = 8$, as per Ref. [28].

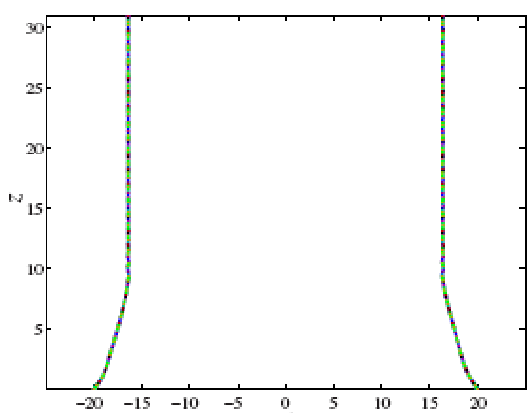

Fig. 6. An example of the collision between two identical slowly moving rhombic vortices, with relative velocity $\Delta V = 2$ and propagation constant μ=22, as per Ref. [28].

4. Conclusions

We have presented a short survey of recent theoretical results produced by studies of spatiotemporal semi-discrete vortical solitons in the model built as a bundle of nonlinear optical fibers with the anomalous group-velocity dispersion, which form a square-shaped lattice in the transverse plane. Stability regions were found for on- and off-site localized patterns with intrinsic vorticity $S = 1$, which are shaped, respectively, as rhombuses and squares, and for quadrupoles, while all vortices with $S \geq 2$ are unstable. Also investigated in detail were collisions between co-axial vortical solitons and/or quadrupoles. Basic outcomes of inelastic and quasielastic collisions were identified.

Acknowledgment

This work was supported, in a part, by a grant from the Israeli-French High Council for Scientific and Technological Research.

References

[1] D. N. Christodoulides, F. Lederer, and Y. Silberberg, Nature (London) **424**, 817 (2003).
[2] F. Lederer, G.I. Stegeman, D.N. Christodoulides, G. Assanto, M. Segev, and Y. Silberberg, Phys. Rep. **463**, 1 (2008).
[3] S. Flach and A.V. Gorbach, Phys. Rep. **467**, 1 (2008).
[4] Y.S. Kivshar, Laser Phys. Lett. **5**, 703 (2008).
[5] P.G. Kevrekidis, K.Ø. Rasmussen, and A.R. Bishop, Int. J. Mod. Phys. B **15**, 2833 (2001).
[6] D. N. Christodoulides and R.I. Joseph, Opt. Lett. **13**, 794 (1988).
[7] M. A. Porter, R. Carretero-González, P. G. Kevrekidis, B. A. Malomed, Chaos **15**, 015115 (2005).
[8] B. A. Malomed, P. G. Kevrekidis, Phys. Rev. E **64**, 026601 (2001).
[9] D. N. Neshev, T. J. Alexander, E. A. Ostrovskaya, Y. S. Kivshar, H. Martin, I. Makasyuk, Z. Chen, Phys. Rev. Lett. **92**, 123903 (2004).
[10] J. W. Fleischer, G. Bartal, O. Cohen, O. Manela, M. Segev, J. Hudock, and D.N. Christodoulides, Phys. Rev. Lett. **92**, 123904 (2004).
[11] P. G. Kevrekidis, B. A. Malomed, Z. Chen, D. J. Frantzeskakis, Phys. Rev. E **70**, 056612 (2004).
[12] H. Sakaguchi and B.A. Malomed, Europhys. Lett. **72**, 698 (2005); R. Driben and B.A. Malomed, Eur. Phys. J. D **50**, 317 (2008).
[13] A. Szameit, J. Burghoff, T. Pertsch, S. Nolte, A. Tünnermann, and F. Lederer, Opt. Express **14**, 6055 (2006); A. Szameit, T. Pertsch, F. Dreisow, S. Nolte, A. Tünnermann, U. Peschel, and F. Lederer, Phys. Rev. A **75**, 053814 (2007).
[14] P. Xie, Z.-Q. Zhang, and X. Zhang, Phys. Rev. E **67**, 026607 (2003); A. Ferrando, M. Zacarés, P. F. de Cordoba, D. Binosi, J.A. Monsoriu, Opt. Express **11**, 452 (2003); A. Ferrando, M. Zacarés, M.A. García-March, Phys. Rev. Lett. **95**, 043901 (2005).
[15] B. A. Malomed, D. Mihalache, F. Wise, L. Torner, J. Opt. B: Quantum Semiclassical Opt. **7**, R53 (2005).
[16] A.B. Aceves, C. De Angelis, A.M. Rubenchik, S.K. Turitsyn, Opt. Lett. **19**, 329 (1994); A.V. Buryak N. N. Akhmediev, IEEE J. Quantum Electron. **31**, 682 (1995).
[17] C.J. Benton, A.V. Gorbach, and D.V. Skryabin, Phys. Rev. A **78**, 033818 (2008).
[18] A.B. Aceves, G.G. Luther, C. De Angelis, A.M. Rubenchik, and S.K. Turitsyn, Phys. Rev. Lett. **75**, 73 (1995).
[19] A.B. Aceves, C. De Angelis, T. Peschel, R. Muschall, F. Lederer, S. Trillo, and S. Wabnitz, Phys. Rev. E **53**, 1172 (1996).
[20] A. B. Aceves, C. De Angelis, G.G. Luther, A. M. Rubenchik, Opt. Lett. **19**, 1186 (1994); E.W. Laedke, K.H. Spatschek, S.K. Turitsyn, V.K. Mezentsev, Phys. Rev. E **52**, 5549 (1995); M. Stepić, L. Hadžievski, M.M. Skorić, Phys. Rev. E **65**, 026604 (2002); A.V. Yulin, D.V. Skryabin, A.G. Vladimirov, Opt. Express **14**, 12347 (2006).
[21] Z.Y. Xu, Y.V. Kartashov, L.C. Crasovan, D. Mihalache, and L. Torner, Phys. Rev. E **70**, 066618 (2004).
[22] D. Mihalache, D. Mazilu, F. Lederer, and Y.S. Kivshar, Opt. Express **15**, 589 (2007); D. Mihalache, D. Mazilu, Y.S. Kivshar, and F. Lederer, *ibid.* **15**, 10718 (2007); D. Mihalache, D. Mazilu, F. Lederer, Y.S. Kivshar, Opt. Lett. **32**, 3173 (2007); D. Mihalache, D. Mazilu, F. Lederer, and Y. S. Kivshar, Phys. Rev. A **79**, 013811 (2009).
[23] D. Mihalache, D. Mazilu, F. Lederer, Y. S. Kivshar, Opt. Lett. **32**, 2091 (2007).
[24] B.B. Baizakov, B.A. Malomed, and M. Salerno, Phys. Rev. A **70**, 053613 (2004); D. Mihalache, D. Mazilu, F. Lederer, Y.V. Kartashov, L.-C. Crasovan, L. Torner, Phys. Rev. E **70**, 055603 (2004); D. Mihalache, D. Mazilu, F. Lederer, B.A. Malomed, L.-C. Crasovan, Y.V. Kartashov, and L. Torner, Phys. Rev. A **72**, 021601 (2005).
[25] H. Leblond, B.A. Malomed, and D. Mihalache, Phys. Rev. A **77**, 063804 (2008).
[26] H. Leblond, B.A. Malomed, and D. Mihalache, Phys. Rev. E **76**, 026604 (2007).
[27] P. G. Kevrekidis, B. A. Malomed, D. J. Frantzeskakis, R. Carretero-González, Phys. Rev. Lett. **93**, 080403 \ (2004); R. Carretero-González, P.G. Kevrekidis, B.A. Malomed and D.J. Frantzeskakis, *ibid*. **94**, 203901 (2005).
[28] H. Leblond, B.A. Malomed, and D. Mihalache, Phys. Rev. A **79**, 033841 (2009).
[29] D. Mihalache, D. Mazilu, F. Lederer, H. Leblond, B. A. Malomed, Phys. Rev. A **77**, 033817 (2008).
[30] D. Mihalache, D. Mazilu, F. Lederer, H. Leblond, B. A. Malomed, Phys. Rev. E **78**, 056601 (2008).

[31] D. Mihalache, D. Mazilu, F. Lederer, H. Leblond, B. A. Malomed, Eur. Phys. J. Special Topics **173**, 245 (2009).
[32] D. Mihalache, D. Mazilu, F. Lederer, Y.S. Kivshar, Phys. Rev. A **79**, 013811 (2009); D. Mihalache, D. Mazilu, F. Lederer, and Y.S. Kivshar, Opt. Commun. **282**, 3000 (2009).
[33] D. Mihalache, D. Mazilu, F. Lederer, Y.S. Kivshar, Phys. Rev. A **77**, 043828 (2008).
[34] D. Mihalache, D. Mazilu, F. Lederer, Y.S. Kivshar, Phys. Rev. E **78**, 056602 (2008).
[35] D. Mihalache, D. Mazilu, F. Lederer, Eur. Phys. J. Special Topics **173**, 255 (2009).
[36] D. Mihalache, D. Mazilu, and F. Lederer, Eur. Phys. J. Special Topics **173**, 267 (2009).
[37] D. Mihalache, D. Mazilu, F. Lederer, Cent. Eur. J. Phys. **8**, 77 (2010); D. Mihalache, D. Mazilu, Rom. Rep. Phys. **60**, 749 (2008); **60**, 957 (2008); **61**, 175 (2009); **61**, 235 (2009); **61**, 587 (2009).

*Corresponding author: herve.leblond@univ-angers.fr