Study of the random crystal field effect on spin-1 and spin-3/2 Blume-Capel models by the position space renormalization group theory

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We study the impact of a random crystal field on magnetic materials of spin-1 and spin-3/2, described by the Blume-Capel model. For this purpose, we use a real space renormalization group approximation, namely the Migdal-Kadanoff one. We give the principal fixed points and phase diagrams found, what allows us to have a better understanding of the critical behavior in these materials. In the two-dimensional case, we observe that randomness, even in small amounts, removes totally the first order phase transitions, replacing them by a smooth continuation and this result is common to spin-1 and spin-3/2 materials; there is only the appearance of second order transitions between the ordered and disordered phases. But at three-dimensions, there is a clear difference between the critical behaviors of spin-1 and spin-3/2 materials. Indeed, for the spin-1 three-dimensional materials, the first order transition is removed, whatever the amount of randomness introduced. On the other hand, for the three-dimensional spin-3/2 materials, the first order transition resists to small amounts of randomness and the first order transition line finishes inside the ordered phase by an end-point. As the randomness increases, the first order transition line becomes shorter and above a certain threshold, defined by a critical value of probability, it disappears completely and is converted into smooth continuation between the ferromagnetic phases.

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1. Introduction

The Blume-Capel model, a spin-1 Ising model with a single anisotropic ion, initially proposed to study the first order magnetic phase transitions, has been investigated by numerous studies and led to many interesting developments [1,2]. Later, the Blume-Emery-Griffiths model [3], used to study the isotopic mixtures of helium ^{3}He-^{4}He and multiple physical systems, was introduced as a generalization of the Blume-Capel model. Several approximation methods, such as the effective field theory [4], the variational methods [5], the Monte Carlo simulations [6], the mean field approximation [7] and the renormalization group techniques [8,9], have been devoted to the study of spin-1 Blume-Capel and Blume-Emery-Griffiths models. One can extend these models by including higher spin values and the spin-3/2 Blume-Capel and Blume-Emery-Griffiths models, proposed to explain the tricritical properties in ternary fluids mixtures [10] and the magnetic and crystallographic phase transitions in some rare-earth compounds such as $DyVO_4$ [11], are probably the simplest extensions, investigated also by several methods such as the mean field approximation [12], the effective field theory [13], the techniques of renormalization group [14,15].

Many studies have been realized in order to have a better understanding of randomness effect [16,17,18]on the phase transitions in physical systems. The randomness can be introduced by the mean of random bonds, random fields or random potential. It has been remarked that whatever the way by which the randomness is introduced, it produces drastic impact on the critical behavior of the systems. Concerning the two-dimensional systems, it is generally observed that the introduction of the most insignificant amount of randomness suffices to remove the first order phase transition, transforming it into a second order one or a smooth continuation. But in systems of upper dimensions, the first order transition generally resists to small amounts of randomness, disappearing completely only above a certain threshold defined by a critical probability. However, for two-dimensional systems as well as for three-dimensional ones, there is not a complete agreement in the results found by using different techniques. The disparity in the results observed shows that the effect of randomness on the phase transitions is not well controlled, given the absence of an exact result.

In our present study, we focus our attention on the spin-1 and spin-3/2 Blume-Capel models, on which we introduce randomness by a crystal field obeying to a bimodal probability distribution. For this purpose, we use the techniques of real space renormalization group, more precisely that of Migdal-Kadanoff [19,20]. The pure version of the spin-1 Blume-Capel model presents a tricritical point between the first and second order lines, which separate the ferromagnetic and paramagnetic phases in both two- and three-dimensional cases. Concerning the spin-3/2 Blume-Capel model, its pure version offers a second order transition line between the ferromagnetic and paramagnetic phases, the two ferromagnetic phases being separated by a first order transition line ending in the second order transition line by a tetracritical point, and such a result is common to the two- and three-dimensional cases. When we introduce the random crystal field, we observe that for the two-dimensional systems, it removes totally the first order phase transition, transforming it into a second order transition one for the spin-1 system and into a smooth continuation for the spin-3/2 system. However, in three dimensions, the spin-1 and spin-3/2 systems react differently to the introduction of randomness; for the spin-1 three-dimensional model, the randomness eliminates the first order transition as in the two-dimensional case, whereas for thespin-3/2 three-dimensional model the first order disappears only after a certain threshold of randomness, determined by a critical value of probability. Below this threshold, the first order transition finishes in the ferromagnetic phases by a critical end-point under the second order transition line. By comparing these results, one notes that there is a dimensional crossover in the critical behavior of the spin-3/2 random Blume-Capel model, while this crossover is absent in the spin-1 case.

The remainder of our present article is organized as follows. We treat in section 2 the formalism of our method and manage to find the Migdal-Kadanoff recursion equations for the spin-1 and spin-3/2 Blume-Capel models. Our principal findings are reported in section 3, where we give different discussions on the fixed points and phase diagrams found and compare our results with those existing in literature.

2. Model and formulations

2.1 Spin-1 case

The spin-1 Blume-Capel model is described by the following Hamiltonian:

$$-\beta H = J\sum_{i,j} S_i S_j - \sum_i \Delta_i S_i^2 \qquad (1)$$

where S_i takes three values, 0, 1 and -1. J and Δ_i represent respectively the reduced bilinear interaction and the crystal field at the site i. The first summation concerns all first nearest neighbor pairs of the lattice and the second one all the sites of a d-dimensional hypercubic lattice.

We will introduce the reduced biquadratic interaction K, albeit it is equal to zero in the Blume-Capel model, in order to have self-consistent recursion relations by the renormalization group technique we adopt. Thus, the Hamiltonian takes the following form:

$$-\beta H = J\sum_{i,j} S_i S_j + K\sum_{i,j} S_i^2 s_j^2 - \sum_i \Delta_i S_i^2 \qquad (2)$$

The crystal field is subject of randomness introduced by a probability distribution of two peaks, that can be written as:

$$P(\Delta_i) = p\delta(\Delta_i + \Delta) + (1-p)\,\delta(\Delta_i - \Delta) \qquad (3)$$

The approximation we use is the Migdal-Kadanoff one, a real space renormalization group technique combining decimation as well as bond shifting. Let us consider a four spins cluster in order to apply the renormalization procedure. We can write the corresponding reduced Hamiltonian as:

$$\begin{aligned} -\beta H = {} & J\left(S_1S_2 + S_2S_3 + S_3S_4\right) \\ & + K\left(S_1^2S_2^2 + S_2^2S_3^2 + S_3^2S_4^2\right) \\ & - \frac{\Delta_1 S_1^2 + \Delta_4 S_4^2}{2d} - \frac{\Delta_2 S_2^2 + \Delta_3 S_3^2}{d} \end{aligned} \qquad (4)$$

After having realized decimation with a spatial rescaling factor $b = 3$ on the two middle spins, one obtains the following Hamiltonian:

$$-\beta\tilde{H} = \tilde{J}S_1S_4 + \tilde{K}S_1^2S_4^2 - \frac{\tilde{\Delta}_1 S_1^2 + \tilde{\Delta}_4 S_4^2}{2d} \qquad (5)$$

where $\tilde{J}$, $\tilde{K}$, $\tilde{\Delta}_1$ and $\tilde{\Delta}_4$ are functions of J, K, Δ_1, Δ_2, Δ_3 and Δ_4 and d.

The renormalization procedure introduces randomness in all the renormalized quantities of the system and leads to a broadening of the parameter space. Thus, the probability distribution of the crystal field, which was of two peaks before renormalization, becomes of eight peaks after. So, to keep the renormalized distribution in the same form as the initial ones, we use the Stinchcombe-Watson [21] approximation and after bond shifting, the Migdal-Kadanoff recursions are formally given by:

$$\begin{cases} J' = b^{(d-1)}\tilde{J} \\ K' = b^{(d-1)}\tilde{K} \\ \Delta_1' = b^{(d-1)}\tilde{\Delta}_1 \end{cases} \qquad (6)$$

2.2 Spin-3/2 case

For the spin-3/2 Blume-Capel model, we need to introduce, in addition to the biquadratic interaction K, two interactions, C and F, added only for a purely technical purpose in order to preserve the parameters space renormalization. Therefore, the spin-3/2 Blume-Capel Hamiltonian will take the following form:

$$-\beta H = J\sum_{i,j} S_i S_j + K\sum_{i,j} S_i^2 s_j^2 + \sum_i \Delta_i S_i^2 + C\sum_{i,j}\left(S_i S_j^3 + S_i^3 S_j\right) + F\sum_{i,j} S_i^3 S_j^3 \quad (7)$$

We use the same probability distribution for the crystal field as in the spin-1 case. The Hamiltonian of a four spins cluster before proceeding the decimation writes:

$$\begin{aligned}-\beta H &= J\left(S_1S_2 + S_2S_3 + S_3S_4\right)\\ &+ K\left(S_1^2S_2^2 + S_2^2S_3^2 + S_3^2S_4^2\right)\\ &+ \frac{\Delta_1 S_1^2 + \Delta_4 S_4^2}{2d} + \frac{\Delta_2 S_2^2 + \Delta_3 S_3^2}{d}\\ &+ C\left(S_1S_2^3 + S_1^3S_2 + S_2S_3^3\right)\\ &+ C\left(S_2^3S_3 + S_3S_4^3 + S_3^3S_4\right)\\ &+ F\left(S_1^3S_2^3 + S_2^3S_3^3 + S_3^3S_4^3\right)\end{aligned} \quad (8)$$

To keep the possible sublattice symmetry breaking character of the system, we perform the decimation by a spatial rescaling factor chosen as an odd integer, $b = 3$. After decimation of the two middle spins, the cluster Hamiltonian is given as follows:

$$-\beta \tilde{H} = \tilde{J}S_1S_4 + \tilde{K}S_1^2S_4^2 + \frac{\tilde{\Delta}_1 S_1^2 + \tilde{\Delta}_4 S_4^2}{2d} + \tilde{C}\left(S_1S_4^3 + S_1^3S_4\right) + \tilde{F}S_1^3S_4^3 \quad (9)$$

with $\tilde{J}$, $\tilde{K}$, $\tilde{\Delta}_1$, $\tilde{\Delta}_4$, $\tilde{C}$ and $\tilde{F}$ the interactions after decimation and functions of J, K, Δ_1, Δ_2, Δ_3, Δ_4, C and F and the dimension d.

As in the spin-1 case, the renormalization procedure introduces randomness in all the physical quantities of the system and leads to a broadening of the parameter space. Using the Stinchcombe–Watson approximation, we obtain after bond shifting the Migdal-Kadanoff recursion equations, too lengthy to be explicitly written here, and expressed as:

$$\begin{cases} J' = b^{d-1}\tilde{J}\\ K' = b^{d-1}\tilde{K}\\ C' = b^{d-1}\tilde{C}\\ F' = b^{d-1}\tilde{F}\\ \Delta_1' = b^{d-1}\tilde{\Delta}_1\end{cases} \quad (10)$$

2. Results and discussions

2.1 Spin-1 and spin-3/2 pure Blume-Capel models

In order to have a better understanding of the random field effect, let us present the results concerning the pure versions of the spin-1 and spin-3/2 Blume-Capel models. To find the results of the pure models, we have taken the probability equal to zero in the Migdal-Kadanoff recursion equation previously established. We find that the two-dimensional systems as well as the three-dimensional ones present the same qualitative critical behavior, differing only by quantitative considerations. Such a consideration is true for the spin-1 and spin-3/2 cases.

In the spin-1 case, we observe two phase transition lines of different critical nature between the ferromagnetic phase and the paramagnetic one. These are the first order transition line occurring at low temperature and the second order line observed at higher temperature, these two lines meeting at a tricritical point. The second order transition line is characterized by a critical fixed point, which coordinates in the parameter space (J, K, Δ) are (0.72,-0.086,+∞,1) in the two-dimensional case and(0.354,-0.0069,+∞,1) in the three-dimensional one.

Regarding the spin-3/2 case, one notes the existence of two second order transition lines between the ordered and disordered phases. The disordered phase is subdivided into two paramagnetic phases of magnetization m=0, one of quadrupolar momentum q>5/4, labeled by $P_{3/2}$, the another of quadrupolar momentum q<5/4, labeled by $P_{1/2}$. The ordered phase is divided into two ferromagnetic phases of magnetization m≠0, denoted by $F_{3/2}$ and $F_{1/2}$, respectively of magnetization q>5/4 and q<5/4. Between the phases $F_{3/2}$ and $P_{3/2}$, is found a second order line, described by the critical fixed point which coordinates in the parameter space (J, K, Δ, C, F), are (0.005,-0.021,+∞,-0.020,0.080) and (1.662,0,+∞,-0.511,0.115)) respectively in the two- and three-dimensional cases. The another second order transition line, characterized by the critical fixed point of coordinates (3.653,-0.021,-∞,-1.622,0.7206) and (1.793,-0.0017,-∞,-0.796,0.354) respectively in the two- and three-dimensional cases, separates the phases $F_{1/2}$and $P_{1/2}$. There is a first order transition arising between the ferromagnetic phases, and it ends on the second order transition lines by a tetracritical point. There is no first order transition between the paramagnetic phases and one can represent them by a single phase, denoted by P.

One can consult the references [22]and [23]where are presented the phase diagrams respectively for the spin-1 and the spin-3/2 models, in both two- and three-dimensional cases.

2.2 Spin-1 and spin-3/2 random Blume-Capel models

2.2.1 The two-dimensional case

By taking a value of probability such that 0<p<1 in the Migdal-Kadanoff recursion equations, we introduce randomness in the spin-1 and spin-3/2 two-dimensional Blume-Capel model, and we find that its introduction produces a remarkable effect on the critical behavior. Indeed, in the spin-1 case as well as in the spin-3/2 one, we note that whatever the insignificant amount of randomness introduced, that suffices to remove the first order phase transition; only those of second order occur.

In the spin-1 case, the second order phase transitions emerging under randomness from the first and second order regimes of the pure model are described by the same critical fixed point and this point presents the same coordinates as those of the fixed point describing the second order transition in the pure two-dimensional model. One can conclude therefore that the second order transitions in the pure and random models belong to the same universality class. We give in Figure 1 an example of phase diagram showing the effect of randomness on the spin-1 two-dimensional model.

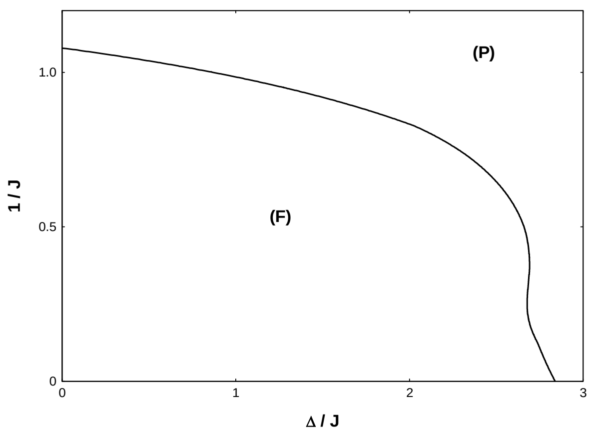

Fig. 1. Phase diagram of the random two-dimensional spin-1 Blume-Capel model in the plane (1/J, Δ/J), probability p=0.14.

In the spin-3/2 case, the first order transition line between the ferromagnetic phases $F_{3/2}$ and $F_{1/2}$,suppressed completely by randomness, is converted into a smooth continuation; then, these two phases can be treated as a single ordered phase, denoted by F. We also remark in this case that the second order transitions belong to the same universality classes as those in the three-dimensional pure model, given that they are described by the same critical fixed point. Fig. 2 is given as an example of phase diagram, plotted in the (1/J, Δ/J) plane for a value of probability p=0.2.

In references [24,25,26], the authors found in agreement with us that the presence of randomness, even in weak amounts, suppresses totally the first order transition, converting it into a continuous one; such a result is what one finds commonly reported in literature. However, let us mention that certain authors, for example in references [27,28], reported contrary results, in which the first order in two-dimensional systems disappears only below a threshold of randomness.

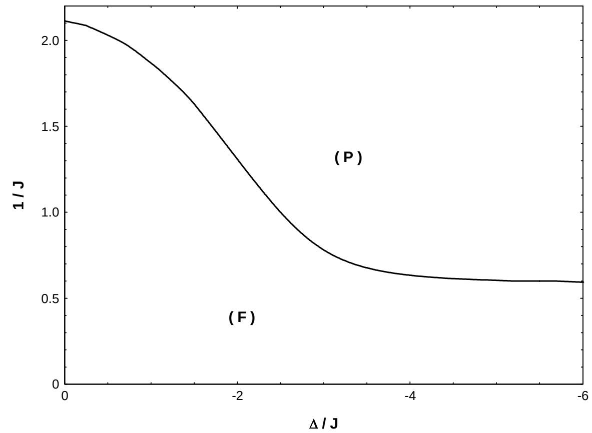

Fig. 2. Phase diagram of the random two-dimensional spin-3/2 Blume-Capel model in the plane (1/J, Δ/J)corresponding to the probability p=0.2.

2.2.2 The three-dimensional case

Contrary to the two-dimensional case, the three-dimensional spin-1 and spin-3/2 models react differently under the influence of randomness. Indeed, the spin-1 model continues to behave like in its two-dimensional case, the first order transition being completely suppressed by randomness and being replaced by a second order transition. One can see it in Figure 3, plotted for a probability p=0.2, but this phase diagram is qualitatively valid for all values of probability.

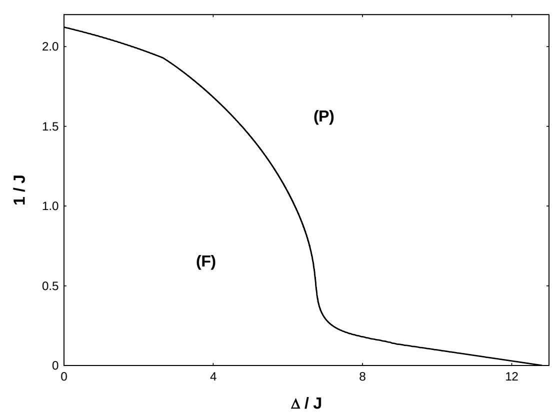

Fig. 3. Phase diagram of the random three-dimensional spin-1 Blume-Capel model in the plane (1/J, Δ/J) corresponding to the value of probability p=0.2.

Regarding the spin-3/2 model, we remark that the first order line resists to small amounts of randomness and finishes in the ordered phase by a critical end-point. Indeed, it has been remarked that the effect of randomness generally decreases when the dimension increases. This is certainly what explains that in the spin-3/2 two-dimensional model, randomness suppresses completely the first order transition whereas the latter remains for small amounts of randomness in the three-dimensional model.

As the randomness increases, the first order line becomes shorter until it reaches the Δ/J axis at a critical probability p_c=0.08, above which the first order no longer exists and is converted into a smooth continuation between $F_{3/2}$ and $F_{1/2}$. So, these phases can be treated at this level by a single ferromagnetic phase, denoted by F. We give in Figs. 4 and 5 the phase diagram for respectively p=0.03, in which the first order is present, and for p=0.1, in which it is absent.

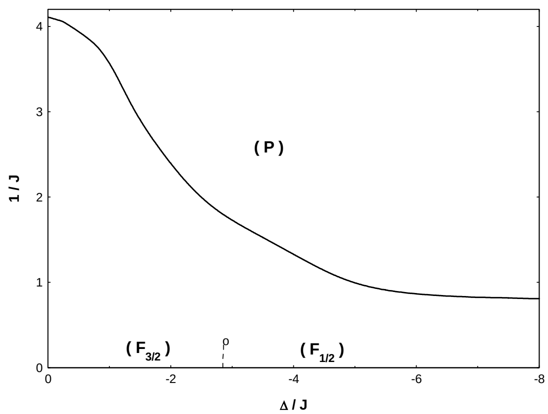

Fig. 4. Phase diagram of the random three-dimensional spin-3/2Blume-Capel model in the plane (1/J, Δ/J) for a probability p=0.03

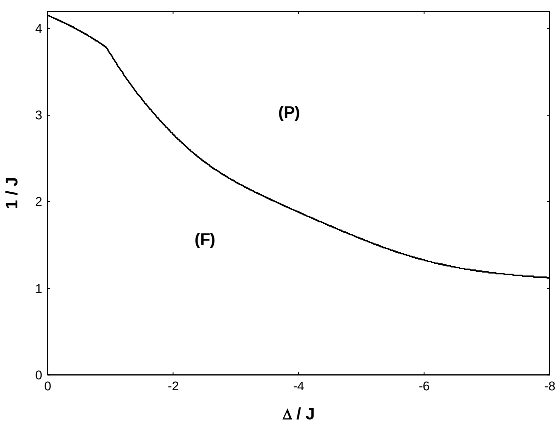

Fig. 5: Phase diagram of the random three-dimensional spin-3/2 Blume-Capel model in the plane (1/J, Δ/J) for a probability p=0.1.

In the two random three-dimensional models, the second order phase transitions belong to the same universality classes as those of the pure models. One will also note that the random spin-3/2 Blume-Capel model presents a dimensional crossover, given that it behaves differently in two and three dimensions, unlike the spin-1 random model.

Our result concerning the absence of first order transition in the three-dimensional random spin-1 model is in agreement with the reference [29], a study on the three-dimensional random field Ising magnet. But it is in disagreement with reference [30] based on a site-diluted four states Potts model. In reference [31], the author, using the Bethe lattice recursions, presents a result similar to our three-dimensional spin-3/2 one, unlike the reference [28] based on the pair approximation, in which the first order transition is always present, whatever the value of probability. The disparity in the results observed shows that the effect of randomness on the phase transitions is still controversial, the nature of the transitions being subject of many uncertainties due to the absent of an exact result.

3. Conclusion

During this study, we have used the Migdal-Kadanoff renormalization group approach to investigate the impact of a random crystal field obeying to a bimodal probability distribution on the spin-1 and spin-3/2 Blume-Capel models on hypercubic lattices. We managed to determine the principal fixed points and phase diagrams, which revealed that the randomness has a marked influence on the critical behavior of the spin systems. Indeed, in the two-dimensional models, the first order transitions have been totally removed and replaced by a second order transition or a smooth continuation, respectively for the spin-1 and spin-3/2 models. Regarding the three-dimensional models, they react differently under randomness, given that the spin-1 model continues to behave like its two-dimensional version, whereas the spin-3/2 model presents a critical probability distinguishing two different critical behaviors, the former occurring for values of probability less than p_c and characterized by the first order transition presence finishing by a critical end-point, the latter concerning values of probability greater than p_c and in which the first order transition is absent. In the random spin-1 and spin-3/2 models, the second order transitions are of the same universality classes as those occurring in the pure models, for two-dimensional case as well as for three-dimensional one.

References

[1] M. Blume, Phys. Rev. **141**, 517 (1966).
[2] H. W. Capel, Physica **32**, 966 (1966).
[3] M. Blume, V. J. Emery, R. B. Griffiths, Phys. Rev. A **4**, 1071 (1971).
[4] Y. Yüksel, Ü Akıncı, H. Polat, Physica A **391,** 2819 (2012),.
[5] C. Buzano, A. Pelizzola, Physica A **216**, 158 (1995).
[6] A. Malakis, A. NihatBerker, N. G. Fytas, T. Papakonstantinou, Phys. Rev. E **85**, 061106 (2012).
[7] H. Ez-zahraouy, A. Kassou-ou-ali, Phys. Rev. B **69**, 064415 (2004).
[8] A.N. Berker, M. Wortis, Phys. Rev. B **14**, 4946 (1986).
[9] S.M. de Oliveira, P.M.C. de Olivera, F.C. de Sa Barreto, J. Stat. Phys. **78**, 1619 (1995).
[10] S. Krinsky, D. Mukamel, Phys. Rev. B **11**, 399 (1975).
[11] J. Sivardière, M. Blume, Phys. Rev. B **5,** 1071 (1972).

[12] L. Bahmad, A. Benyoussef, A. El Kenz, J. Magn. Magn.Mater.**320**, 397 (2008).
[13] T. Kaneyoshi, M. Jascur, Phys. Lett. A **177**, 172 (1993).
[14] A. Saguia, B. Boechat, M.A. Continentino, Phys. Rev. B **68**, 020403 (2003).
[15] A. Bakchich, M. El Bouziani, Phys. Rev. B **63**, 064408 (2001).
[16] S. Fishman, A. Aharony, J. Phys. C **12**, 729 (1979).
[17] Y. Imry, S. K. Ma, Phys. Rev. Lett.**35**, 1399 (1975).
[18] M. El Yadari, M. R. Benayad, A. Benyoussef, A. El Kenz, Physica A **389**, 4677 (2010).
[19] A. A. Migdal, Zh. Eksp. Teor.Fiz. **69**, 810 (1975) [Sov.Phys. JETP 42, (1975) 743].
[20] L. P. Kadanoff, Ann. Phys. (N.Y.) **100**, 359 (1976).
[21] R. B. Stinchcombe, B.P. Watson, J. Phys. C: Solid State Phys. **9**, 3221 (1976).
[22] A. Benyoussef , N. Boccara , M. El Bouziani, Phys. Rev. B **34**, 7775 (1986).
[23] M. El Bouziani, A. Gaye, Physica A, doi: 10.1016/j.physa.2013.02.003 (2013).
[24] N. S. Branco, B. M. Boechat, Phys. Rev. B **56**, 11673 (1997).
[25] A. Malakis, A. NihatBerker, I. A. Hadjiagapiou, N. G. Fytas, Physical Review E **79**, 011125 (2009).
[26] M. El Bouziani, A. Gaye, A. Jellal, Physica A **392**, 689 (2013).
[27] G. M. Zhang, C.Z. Yang, Acta Phys. Sin. **42**, 128 (1993).
[28] D. P. Lara, J. A. Plascak, Physica **A260** 443 (1998).
[29] A. A. Middleton and D. S. Fisher, Phys. Rev. B **65**, 134411 (2002).
[30] L. A. Fernández, A. Gordillo-Guerrero, V. Martín-Mayor, J. J. Ruiz-Lorenzo, Phys. Rev. Lett. **100**, 057201 (2008)
[31] E. Albayrak, J. Magn. Magn.Mater.**323**, 2846 (2011).

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