

# Temperature, stress, and rate dependent numerical implementation of magnetization processes in phenomenological models

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In this article we present advances in the numerical analysis of temperature, stress, and rate dependent magnetization processes that we have recently implemented in the framework of phenomenological simulator for hysteretic modeling–HysterSoft. Our implementation allows one to make a detailed comparison of various models of hysteresis and to establish the limits of applicability of each model. Special emphasis is given to presenting the numerical algorithms for the description of the magnetization processes (e.g. rate dependent processes) in a “universal” way which makes them suitable for the implementation of our algorithms in the framework of any model of hysteresis, such as the Preisach, Energetic, and Jiles-Atherton models.

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## 1. Introduction

Magnetic hysteresis is one of the most important properties of magnetic materials. In many applications, such as magnetic recording and magnets, hysteresis is a desired phenomenon on which the functioning of the whole device is based. In other applications, such as transformers and inductances hysteresis is unwanted phenomenon which should be avoided. In both cases, the magnetic hysteresis phenomena should be very well characterized and modeled in order to be able to account for them in the design of the device. Whereas there is much effort in the literature for the analysis of hysteresis in magnetic materials in normal external conditions [1-4], there is little effort to characterize and model the magnetic hysteresis in extreme conditions such as high mechanical stress, high and low temperatures and frequencies. In this work we give the basic equations and present advances in the numerical analysis of magnetic hysteresis that were recently implemented in HysterSoft [5]. The advantage of our implementation method is that it allows real time simulations of devices and circuits containing hysteretic materials.

This article is structured as follows. In Section II we present the relaxation time and effective field approximations for the description of rate-dependent magnetic processes. In Section III we present our model for the description of stress dependent processes in magnetic materials. Section IV is devoted to the analysis of temperature dependent magnetization processes, which is followed by conclusions.

## 2. Rate dependent magnetic processes

Let us consider a general model of hysteresis (such as the Preisach model, Jiles-Atherton model, Energetic model, Hodgdon model, etc.), in which magnetization  $M(t)$  can be written as a function of the effective magnetic field  $H_{eff}(t)$  as:

$$M(t) = \hat{\Gamma} H_{eff}(t), \quad (1)$$

where  $\hat{\Gamma}$  is the hysteresis operator. In the following we define two dynamic models, first is based on the mean field approximation and the second on the relaxation time approximations. In the mean field the approximation effective field is given by [6,7]:

$$H_{eff} = H + F(M, \dot{M}), \quad (2)$$

where  $F$  is a function of magnetization  $M$  and of its derivative with respect to time,  $\dot{M}$ . Equations (1) and (2) represent a system of nonlinear, coupled equations that is to be solved for magnetization  $M$ . In the framework of the Preisach model, system is a system of integral-differential equations, while in the framework of the Jiles, Energetic and Hodgdon models it is a system of differential equations. It should be noted that, in the framework of the moving models of hysteresis, the effective magnetic field can be written as

$H_{eff} = H + \alpha M$  where  $\alpha$ , is the moving parameter. Thus, our model can be regarded as a generalization of the moving model with  $F(M, \dot{M}) = \alpha M$ .

In the relaxation time approximation, magnetization  $M(t)$  can be described by the following first-order differential equation:

$$\frac{dM}{dt} = -\frac{M(t) - M_\infty(t)}{\beta}, \quad (3)$$

where  $\beta$  is a relaxation time parameter. The magnetic susceptibility can be computed by using:

$$\chi(t) = -\frac{M(t) - M_\infty(t)}{\beta \dot{H}(t)}, \quad (4)$$

where  $\dot{H}(t)$  is the derivative of the applied magnetic field with respect to time. Equations (1), (3), and (4) have to be solved to compute magnetization  $M$  as a function of applied field  $H(t)$ . In our simulations we integrate them numerically by using the finite difference approximation. In Fig. 1 we present sample magnetic simulations obtained by using the two approximations for a barium ferrite.

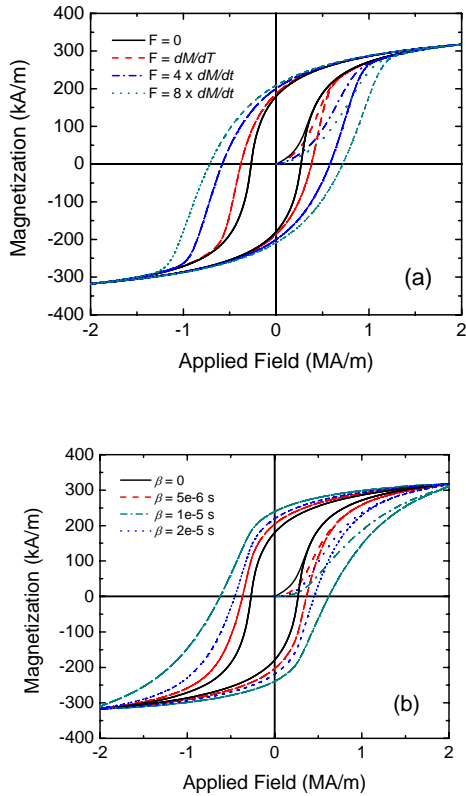


Fig. 1. Dynamic hysteresis loops for a barium ferrite obtained in the framework of the Energetic model by using the effective field (a) and relaxation time (b) approximations.

### 3. Stress dependent magnetic processes

Stress dependent magnetic processes are considered in the framework of the Energetic model [1]. If we denote by  $k_0$ ,  $q_0$ ,  $g_0$ , and  $h_0$  the parameters of the Energetic model under no stress, we can first compute the following parameters:

$$c_k = \frac{k_0}{\lambda_s^2 E_Y}, \quad c_q = q_0 \frac{K_1}{k_0}, \quad c_g = g_0 \frac{\mu_0 M_s^2}{K_1}, \quad (5)$$

$$c_h = \frac{h_0}{M_s} (c_r + 1) (e^{g_0 \ln^2} - 1) + \frac{c_k \lambda_s^2 E_Y}{\mu_0 M_s^2} + N_e,$$

where  $E_Y$  is the Young constant,  $K_1$  is the magnetic anisotropy constant, and  $\lambda_s$  is the magnetoscription constant at saturation. Then we compute the model parameters under stress level  $\sigma$  (measured in Pa):

$$k = c_k \lambda_s^2 E_Y, \quad q = c_q \frac{2K_1 + 3(1 + \nu_p \lambda_s) \sigma}{2c_k \lambda_s^2 E_Y}, \quad (6)$$

$$c_g = \frac{2K_1 + 3(1 + \nu_p) \lambda_s \sigma}{2\mu_0 M_s^2},$$

$$h = \frac{\mu_0 M_s^2 (c_h - N_e) - c_k \lambda_s^2 E_Y}{\mu_0 M_s (c_r + 1) (e^{g \ln^2} - 1)} \quad (7)$$

where  $\nu_p$  is the Poisson ratio (for permalloy  $\nu_p = 0.32$ ).

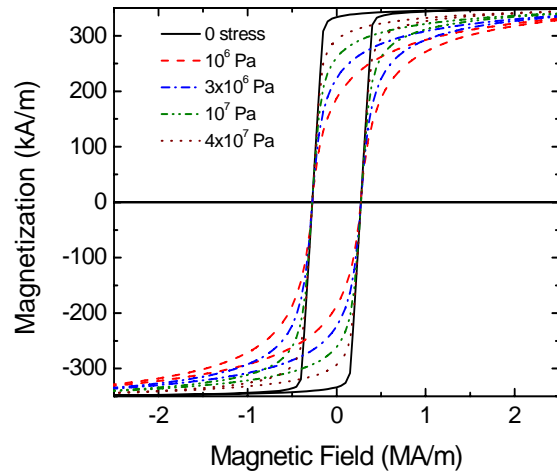


Fig. 2. Mechanic stress effects on a barium ferrite.

The effects of mechanical stress on the major hysteresis loop of a barium ferrite are shown in Fig. 2 for  $10^6$  Pa,  $3 \times 10^6$  Pa,  $10^7$  Pa, and  $4 \times 10^7$  Pa. In Figs. 3 we present the effects of stress and frequency on the same material when the magnetic field is varied time according to the inset on these figures. No stress and no dynamic effects are considered in the simulations presented in Fig. 2(a). No stress is assumed in Fig. 3(b) while  $\sigma = 4 \times 10^7$  Pa is applied on the magnetic material in the simulations presented in Fig. 3(c). The total computation time for the simulations presented in Figs. 3 is less than one second on a 2 GHz one processor computer, which makes our model very attractive for real time simulations of stress induced effects in magnetic circuits.

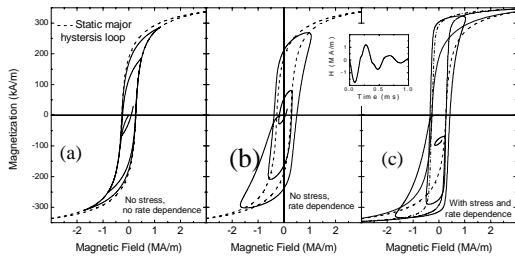


Fig. 3. Barium ferrite under stress and high rates of variation of the applied magnetic field. The magnetic field is represented in the inset as a function of time.

#### 4. Temperature dependent magnetic processes

Temperature dependent simulations are also considered in the framework of the Energetic model [1]. The magnetization of saturation depends on temperature  $T$  as:

$$M_s(T) = M_{s0} \left( 1 + \frac{T}{2T_C} \right) \sqrt{1 - \frac{T}{T_C}}, \quad (8)$$

where  $M_{s0}$  is the magnetization of saturation at  $T = 0$  and  $T_C$  is the Curie temperature. The anisotropy constant and  $\lambda_s$  are assumed to depend on the temperature as:

$$K_1(T) = K_1(T_0) [1 + \alpha_K (T - T_0)] \quad (9)$$

and

$$\lambda_s(T) = \lambda_s(T_0) [1 + \alpha_\lambda (T - T_0)], \quad (10)$$

respectively, where  $\alpha_K$  and  $\alpha_\lambda$  are constants that should be identified at  $T_0 = 300$  K. The effects of the temperature on the barium ferrite are shown in the simulations presented in Fig. 4. The total area of the magnetic hysteresis is decreased due the lowering of the magnetization at saturation and the decrease in the anisotropy constant of the barium ferrite, which results in the decrease of the coercive field.

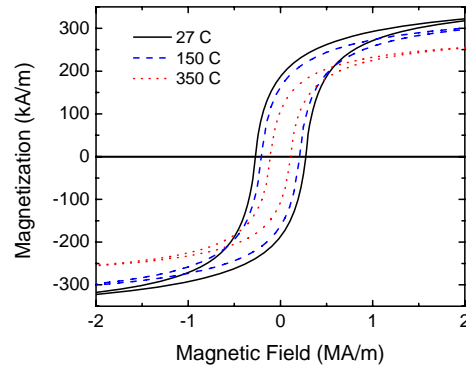


Fig. 4. Temperature effects on a barium ferrite.

#### 4. Conclusions

An efficient implementation of temperature, stress, and rate dependent magnetization processes is implemented for phenomenological models of hysteresis. Such models include the Jiles-Atherton, Energetic, Preisach, and Hodgdon models. Rate dependent processes are implemented by using the relaxation time and effective field approximations, while the temperature and mechanical stress analysis can be done in the framework of the Energetic model.

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