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# The dynamics of screening spatial solitons in photorefractive-photovoltaic crystals incorporating higher order space charge fields

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In the present paper, we have studied the dynamics of two spatially separated screening optical solitons in photorefractive-photovoltaic materials incorporating higher order diffusion phenomenon. We have employed paraxial ray approximation method to derive evolution equations of different parameters which characterize the dynamics of two interacting soliton. This approach yields a system of coupled ordinary differential equations for evaluation of solitons parameters such as spatial width and centre of gravity, and subsequently employed numerical method to extract information on their dynamics. Oscillatory stationary bound states predicted, though no stable composite bound state exists.

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## 1. Introduction

During last three decades, optical solitons have been extensively studied [1-19] topic not only due to their mathematical elegance and but also due to the possibility of applications. They are self guided waves and are able to maintain their shape while propagating. These solitons are characterized as spatial, temporal or spatio-temporal depending on their localization in space, time or both in space and time. Spatial solitons are non-diffracting optical beams which maintain their shape owing to the cancellation of diffraction by the optical nonlinearity of the medium in which they are propagating. Though solitons have been detected in almost all branches of physics, spatial photorefractive solitons possesses some unique properties which make them attractive in several applications such as all optical switching and routing, interconnects [2-4].

To date, three different types of steady state photorefractive solitons have been predicted. Photorefractive screening solitons is the one which was identified first. In the steady state, both bright and dark screening solitons (SS) are possible when an external bias voltage is appropriately applied to a non-photovoltaic photorefractive crystal [20-22]. The second kind is the photovoltaic soliton [23-24], the formation of which however requires an unbiased PR crystal that exhibits photovoltaic effect. The third type of photorefractive soliton arises when an electric field is applied to a photovoltaic photorefractive crystal [25, 26]. These solitons owe their existence to both photovoltaic effect and spatially non-uniform screening of the applied field and are also known as screening photovoltaic (SP) soliton.

In a photorefractive crystal, both drift and diffusion processes are responsible for the creation of space charge field. This space charge field is responsible for the change in refractive index of the crystal which in turn is responsible for the formation of soliton. When biasing field is strong, the diffusion process does not contribute significantly to the formation of solitons. Several authors, however, have studied soliton dynamics incorporating diffusion process, albeit with approximation [27, 28]. In this paper we study the existence of bright screening photovoltaic spatial solitons through photorefractive crystals incorporating higher order diffusion processes and also obtain the equation for the trajectory of the soliton. The arrangement of the paper is as follows: The mathematical model for soliton propagation has been formulated and developed in Section 2. Section 3 includes the results and discussions. A brief conclusion has been presented in Section 4.

## 2. Mathematical model

To start with, we consider a pair of optical beams which are propagating in a biased photorefractive-photovoltaic crystal along z-direction. They are of same frequency but mutually incoherent. The crystal is taken to be $LiNbO_3$ with its optical c-axis oriented along the x coordinate. The two optical beams are allowed to diffract only along the x-direction and for the sake of simplicity the photorefractive crystal is assumed to be loss less. We assume that the incident soliton forming optical beams are polarized along the x direction. The optical fields of two soliton forming optical beams are expressed

$$\vec{E}_1 = \hat{x}\phi_1(x,z)\exp(ikz) \quad \text{and}$$
$$\vec{E}_2 = \hat{x}\phi_2(x,z)\exp(ikz) \qquad (1)$$

Where, $k = k_0 n_e = \left(\frac{2\pi}{\lambda_0}\right) n_e$, $n_e$ is the unperturbed extraordinary index of refraction and $\lambda_0$ is the free space wavelength; $\phi_1$ and $\phi_2$ are slowly varying envelopes of two optical fields, respectively . It can be readily shown that [6] the slowly varying envelopes of two interacting spatial solitons inside the photovoltaic PR crystal are governed by the following evolution equations:

$$i\frac{\partial\phi_1}{\partial z} + \frac{1}{2k}\frac{\partial^2\phi_1}{\partial x^2} - \frac{k_0 n_e^3 r_{33} E_{SC}}{2}\phi_1(x,z) = 0 \qquad 2(a)$$

$$i\frac{\partial\phi_2}{\partial z} + \frac{1}{2k}\frac{\partial^2\phi_2}{\partial x^2} - \frac{k_0 n_e^3\ r_{33} E_{SC}}{2}\phi_2(x,z) = 0 \qquad 2(b)$$

Where, $r_{33}$ is the electro-optic coefficient, and $E_{SC}$ is the space charge field which perturbs the refractive index through the Pockel's effect. For convenience, we transform envelope equations into normalized equations by the following substitutions:

$$\xi = \frac{z}{kx_0^2}, \quad \phi_1 = \sqrt{\frac{2\eta_0 I_d}{n_e}}\ U_1, \quad \phi_2 = \sqrt{\frac{2\eta_0 I_d}{n_e}}\ U_2$$

, $k = k_0 e$ and $s = \frac{x}{x_0}$. By virtue of these substitution, we get following normalized equation

$$i\frac{\partial U_1}{\partial\xi} + \frac{1}{2k}\frac{\partial^2 U_1}{\partial s^2} - \frac{k_0^2 x_0^2 n_e^4\ r_{33} E_{SC}}{2} U_1 = 0 \quad 3(a)$$

$$i\frac{\partial U_2}{\partial\xi} + \frac{1}{2k}\frac{\partial^2 U_2}{\partial s^2} - \frac{k_0^2 x_0^2\ n_e^4\ r_{33} E_{SC}}{2} U_2 = 0 \quad 3(b)$$

The induced space charge field $E_{SC}$ can be obtained from the standard set of rate and continuity equations and Gauss's law, which turns out to be [29]:

$$E_{sc} = E_{sco}\left(1 + \frac{\varepsilon_0\varepsilon_r}{eN_A}\frac{\partial E_{sc}}{\partial x}\right) - \frac{K_B T}{e}\ \frac{1}{I + I_d}\ \frac{\partial I}{\partial x} + \frac{K_B T}{e}\ \frac{\varepsilon_0\varepsilon_r}{eN_A}\ \frac{\partial^2 E_{sc}}{\partial x^2} \qquad (4)$$

$$E_{sco} = E_0\frac{I_d}{I + I_d} - E_p\frac{I}{I + I_d}, \qquad (5)$$

where $E_{SC}$ is the first order of $E_{SC}$ , $I = I(x,z)$ is the intensity of the optical beam, $I_d$ is the so-called dark irradiance, $N_A$ is the acceptor density, $E_0$ is the strength of bias field , $E_p = \frac{k_p \nu_R N_A}{e\mu}$ is the photovoltaic filed constant, $\nu_R$ is the carrier recombination rate, $\mu$ and $e$ are, respectively, the electron mobility and the charge, $k_p$ is the photovoltaic constant, $K_B$ is Boltzmann's constant, $T$ is the absolute temperature, $\varepsilon_0$ is the free space permittivity, and $\varepsilon_r$ as the relative static dielectric constant. $E_{SCO}$ is obtained when diffusion is ignored and the dimensionless term $\left|\left(\frac{\varepsilon_0\varepsilon_r}{eN_A}\right)\left(\frac{\partial E_{sc}}{\partial x}\right)\right|$ is much less than unity, this condition fulfills in typical photovoltaic photorefractive media if power density $I(x,z)$ of the optical beam varies slowly with respect of $x$ .

To study the effects those arise from higher order terms such as $\frac{\partial E_{sc}}{\partial x}$ and $\frac{\partial^2 E_{sc}}{\partial x^2}$ in Eq. (4), we now use $E_{SC0}$ as the first order in $E_{SC}$ and expand $E_{SC}$ with terms of first and higher orders. Therefore, the perturbative solution of the space charge field $E_{SC}$ reads as follows:

$$E_{SC} = E_{SC0} + E_\nu + E_{\nu_1} + E_{\nu_2} + E_{\nu_3} + E_{\nu_4}, \quad (6)$$

where

$$E_\nu = -\frac{K_B T}{e}\ \frac{1}{I + I_d}\ \frac{\partial I}{\partial x}, \qquad 7(a)$$

$$E_{\nu_1} = -\frac{\varepsilon_o\varepsilon_r}{eN_A}\ E_0(E_0 + E_P)\ \frac{I_d^{\ 2}}{(I + I_d)^3}\ \frac{\partial I}{\partial x}, \qquad 7(b)$$

$$E_{\nu_2} = \frac{\varepsilon_o \varepsilon_r}{eN_A} E_0 (E_0 + E_P) \frac{I I_d}{(I + I_d)^3} \frac{\partial I}{\partial x} \quad , \qquad 7(c)$$

$$E_{\nu_3} = 2 \frac{K_B T}{e} \frac{\varepsilon_o \varepsilon_r}{eN_A} (E_0 + E_P) \frac{I_d}{(I + I_d)^3} \left( \frac{\partial I}{\partial x} \right), \quad 7(d)$$

$$E_{\nu_4} = \frac{K_B T}{e} \frac{\varepsilon_o \varepsilon_r}{eN_A} (E_0 + E_P) \frac{I_d}{(I + I_d)^2} \left( \frac{\partial^2 I}{\partial x^2} \right) \quad 7(e)$$

It is important to note that Eq. (6) is valid as long as the perturbations $E_\nu$ and $E_{\nu_i}$ $(i = 1,2,3,4)$ are much smaller than the leading terms of the space charge field $E_{SC0}$. We can now establish the envelope evolution equation by substituting the expression for the perturbed refractive index into the paraxial wave equation. After appropriate normalizations and neglecting the crystal loss, the envelopes $U_1$ and $U_2$ are then found to obey the following dynamical evolution equations:

$$i\frac{\partial U_1}{\partial \xi} + \frac{1}{2}\frac{\partial^2 U_1}{\partial s^2} - \beta \frac{U_1}{1+|U_1|^2+|U_2|^2} + \alpha \frac{|U_1|^2+|U_2|^2}{1+|U_1|^2+|U_2|^2} U_1$$
$$+ \nu \frac{\left(|U_1|^2+|U_2|^2{}_s\right)}{1+|U_1|^2+|U_2|^2} U_1 + \nu_1 \frac{[\left(|U_1|^2+|U_2|^2\right)_s]}{(1+|U_1|^2+|U_2|^2)^3} U_1 - \nu_2 \frac{\left(|U_1|^2+|U_2|^2\right)_s \left(|U_1|^2+|U_2|^2\right)}{(1+|U_1|^2+|U_2|^2)^3} U_1$$

$$- \nu_3 \frac{[\left(|U_1|^2+|U_2|^2\right)_s]^2}{(1+|U_1|^2+|U_2|^2)^3} U_1 + \nu_4 \frac{[|U_1|^2+|U_2|^2{}_{ss}]}{1+\left(|U_1|^2+|U_2|^2\right)^2} U_1 = 0 \quad , \qquad (8)$$

and

$$i\frac{\partial U_2}{\partial \xi} + \frac{1}{2}\frac{\partial^2 U_2}{\partial S^2} - \beta \frac{U_2}{1+|U_1|^2+|U_2|^2} + \alpha \frac{|U_1|^2+|U_2|^2}{1+\ |U_1|^2+|U_2|^2} U_2$$
$$+ \nu \frac{[(|U_1|^2+|U_2|^2)_s]}{1+|U_1|^2+|U_2|^2} U_2 + \nu_1 \frac{[(|U_1|^2+|U_2|^2)_s]}{(1+|U_1|^2+|U_2|^2)^3} U_2 - \nu_2 \frac{[(|U_1|^2+|U_2|^2)_s[|U_1|^2+|U_2|^2]}{(1+|U_1|^2+|U_2|^2)^3} U_2$$
$$- \nu_3 \frac{[|U_1|^2+|U_2|^2)_s]^2}{(1+|U_1|^2+|U_2|^2)^3} U_2 + \nu_4 \frac{(|U_1|^2+|U_2|^2)_{ss}}{(1+|U_1|^2+|U_2|^2)^2} U_2 = 0, \qquad (9)$$

where $\chi = \sigma \mathrm{E}_p$, $\beta = \sigma E_0$, $\nu = \sigma \tau$, $\nu_1 = k_1 (\alpha + \beta)$, $\nu_2 = k_2 (\alpha + \beta)$, $\nu_3 = 2\tau S (\alpha + \beta)$, $\nu_4 = \frac{\nu_3}{2}$, $k_1 = SE_0$, $k_2 = SE_P$, $\sigma = \frac{(k_0 x_0)^2 n e^4 r_{33}}{2}$, $\tau = \frac{k_B T}{e x_0}$ and $S = \frac{\varepsilon_0 \varepsilon_r}{E N_A x_0}$; $\nu$ is the first order diffusion terms, $\nu_1$ and $\nu_2$ are the higher order odd perturbations terms, which will have odd effect (such as beam deflection) as $\nu$ does; $\nu_3$ and $\nu_4$ are the higher order even perturbation terms, which produce even effects such as spatial broadening. The sings of $\nu_1$ and $\nu_2$ are determined by the polarity of bias field $E_0$ and photovoltaic field $E_p$, respectively, whereas $\nu_1$, $\nu_3$ and $\nu_4$ are always positive. The sign of $E_p$ depends on the characteristics of the crystal and the polarization of the light.

Equations (8) and (9) are non-integrable, and their exact solutions cannot be obtained. In order to solve such non-integrable equations, various approximation methods have been devised, such as Anderson's variation method [30], the moment method of Vlasov [31] and the paraxial method of Akhmanov [32, 33]. In the present investigation, we employ the paraxial method of Akhmanov to get the solution for the dynamical equations (8) and (9) in the PR medium. We start the following ansatz for the slowly varying envelopes:

$$U_i(\xi,s) = U_{0i}(\xi,s)\exp[-i\Omega_i(\xi,s)] \quad (10)$$

where, $i = 1,2$; $\Omega_i(\xi,s)$ and $U_{0i}(\xi,s)$ are respectively the arbitrary phase and the amplitude of the envelope and these are assumed to be real. Substitution of relationship (10) in equations (8) and (9) yield the following coupled second order differential equations:

$$\frac{\partial U_{0i}}{\partial \xi} - \frac{\partial U_{0i}}{\partial s}\frac{\partial \Omega_i}{\partial s} - \frac{1}{2}\frac{\partial^2 \Omega_i}{\partial s}U_{0i} = 0, \quad (11)$$

$$U_{0i}\frac{\partial \Omega_{0i}}{\partial \xi} + \frac{1}{2}\frac{\partial^2 U_{0i}}{\partial s^2} \quad \frac{1}{2}\quad \frac{\partial \Omega_i}{\partial s}^{2} U_{0i} - \beta\phi_1(\xi,s)U_{0i} + \alpha\phi_2(\xi,s)U_{0i} + \nu\phi_3(\xi,s)U_{0i} +$$
$$+\nu_1\phi_4(\xi,s)U_{0i} \quad \nu_2\phi_5(\xi,s)U_{0i} \quad \nu_3\phi_6(\xi,s)U_{0i} + \nu_4\phi_7(\xi,s)U_{0i}, \quad (12)$$

where,

$$\phi_1 = \frac{1}{1+U_{01}^2+U_{02}^2}, \phi_2 = \frac{\left(U_{01}^2+U_{02}^2\right)}{1+U_{01}^2+U_{02}^2},$$

$$\phi_3 = \frac{\left(U_{01}^2+U_{02}^2\right)^2}{1+U_{01}^2+U_{02}^2}, \phi_4 = \frac{U_{01}^2+U_{02}^2}{1+U_{01}^2+U_{02}^2},$$

$$\phi_5 = \frac{\left(U_{01}^2+U_{02}^2\right)\left(U_{01}^2+U_{02}^2\right)_S}{\left(1+U_{01}^2+U_{02}^2\right)^3}, \phi_6 = \frac{\left[\left(U_{01}^2+U_{02}^2\right)_S\right]^2}{\left(1+U_{01}^2+U_{02}^2\right)^3}$$

and $\phi_7 = \dfrac{\left(U_{01}^2+U_{02}^2\right)_{SS}}{\left(1+U_{01}^2+U_{02}^2\right)^3}$.

We look for a self-similar bright spatial soliton solution for which the field energy is confined in the central region of the beam, thus we take

$$U_{0i} = \frac{U^2{}_{00i}}{f_i}\ \exp\left[\frac{-(s-s_{0i})^2}{r_0^2 f_i^2}\right], \quad (13)$$

$$\Omega_i(\xi,s) = \frac{(s-s_{oi}(\xi))^2}{2}\ \delta_i(\xi) - (s-s_{oi}(\xi))\frac{\partial s_{oi}}{\partial \xi} + \psi_i(\xi), \quad (14)$$

$$\delta_1(\xi) = -\frac{i}{f_i}\frac{\partial f_i}{\partial \xi}, \quad i = 1,2, \quad (15)$$

where, $U_{001}$, and $U_{002}$, respectively represent the peak power of these solitons. $r_0$ is a positive constant, $f_1(\xi)$ and $f_2(\xi)$ are normalized beam width parameters, $r_0 f_1$, and $r_0 f_2$ are respectively, the spatial widths of these solitons. $S_{01}(\xi)$ and $S_{02}(\xi)$ are position of the center of two solitons, respectively, and $\psi_1(\xi)$ and $\psi_2(\xi)$ are the longitudinal phase associated with these solitons. The nonlinear contributions $\phi_1(\xi,s)$ through $\phi_7(\xi,s)$ to the refractive index are expanded in the Taylor series around the beam centre. Under second –order approximation we obtain:

$$\phi_1(\xi,s) = \frac{1}{1+\frac{U_{001}^1}{f_1}+\frac{U_{002}^2}{f_2}} + (s-s_{01})^2\frac{\left(\frac{U_{001}^2}{r_0^2 f_1^3}+\frac{U_{002}^2}{r_0^2 f_2^3}\right)}{\left(1+\frac{U_{001}^2}{f_1}+\frac{U_{002}^2}{f_2}\right)} + (s-s_{02})^2\frac{\left(\frac{U_{001}^2}{r_0^2 f_1^3}+\frac{U_{002}^2}{r_0^2 f_2^3}\right)}{\left(1+\frac{U_{001}^2}{f_1}+\frac{U_{002}^2}{f_2}\right)} \quad 16\ (a)$$

$$\phi_2(\xi,s) = \frac{\frac{U_{001}^2}{f_1}+\frac{U_{002}^2}{f_2}}{1+\frac{U_{001}^2}{f_1}+\frac{U_{002}^2}{f_2}} +$$
$$(s \quad s_{01})^2 \frac{\frac{U_{001}^2}{r_0^2 f_1^3}+\frac{U_{002}^2}{f_2}}{1+\frac{U_{001}^2}{f_1}+\frac{U_{002}^2}{f_2}} + \frac{\frac{U_{001}^4}{r_0^2 f_1^4}+\frac{U_{001}^2 U_{002}^2}{r_0^2 f_1^3 f_2}+\frac{U_{002}^4}{r_0^2 f_2^4}+\frac{U_{001}^2 U_{002}^2}{r_0^2 f_1 f_2^3}}{1+\frac{U_{001}^2}{f_1}+\frac{U_{002}^2}{f_2}\ ^2} +$$
$$(s \quad s_{02})^2 \frac{\frac{U_{001}^2}{r_0^2 f_1^3}+\frac{U_{002}^2}{f_2}}{1+\frac{U_{001}^2}{f_1}+\frac{U_{002}^2}{f_2}} + \frac{\frac{U_{001}^4}{r_0^2 f_1^4}+\frac{U_{001}^2 U_{002}^2}{r_0^2 f_1^3 f_2}+\frac{U_{002}^4}{r_0^2 f_2^4}+\frac{U_{001}^2 U_{002}^2}{r_0^2 f_1 f_2^3}}{1+\frac{U_{001}^2}{f_1}+\frac{U_{002}^2}{f_2}\ ^2} \quad 16\ (b)$$

$$\phi_3(\xi,s)=(s-s_{01})\left[\frac{\dfrac{-2U_{001}^2}{r_0^2 f_1^3}+\dfrac{-2U_{002}^2}{r_0^2 f_2^3}}{1+\dfrac{U_{001}^2}{f_1}+\dfrac{U_{002}^3}{f_2}}\right]$$

$$-(s-s_{01})\left[\frac{\dfrac{-2U_{001}^2}{r_0^2 f_1^3}+\dfrac{-2U_{002}^2}{r_0^2 f_{002}^3}}{1+\dfrac{U_{001}^2}{f_1}+\dfrac{U_2^3}{f_2}}\right], \qquad 16\text{ (c)}$$

$$\phi_4(\xi,s)=(s-s_{01})\left[\frac{\dfrac{-2U_{001}^2}{r_0^2 f_1^3}+\dfrac{-2U_{002}^2}{r_0^2 f_2^3}}{\left(1+\dfrac{U_{001}^2}{f_1}+\dfrac{U_{002}^2}{f_2}\right)^3}\right]$$

$$-(s-s_{02})\left[\frac{\dfrac{-2U_{001}^2}{r_0^2 f_1^3}+\dfrac{-2U_{002}^2}{r_0^2 f_2^3}}{\left(1+\dfrac{U_{001}^2}{f_1}+\dfrac{U_{002}^2}{f_2}\right)^3}\right], \qquad 16\text{(d)}$$

$$\phi_5(\xi,s)=(s-s_{01})\left[\frac{\dfrac{-2U_{001}^4}{r_0^2 f_1^4}+\dfrac{-2U_{001}^2U_{002}^2}{r_1^3 f_2}+\dfrac{-2U_{002}^4}{r_0^2 f_2^4}+\dfrac{-2U_{001}^2U_{002}^2}{r_0^2 f_1 f_2^3}}{\left(1+\dfrac{U_{001}^2}{f_1}+\dfrac{U_{002}^2}{f_2}\right)^3}\right]$$

$$-(s-s_{01})\left[\frac{\dfrac{-2U_{001}^4}{r_0^2 f_1^4}+\dfrac{-2U_{002}^2U_{001}^2}{r_1^3 f_2}+\dfrac{-2U_{002}^4}{r_0^2 f_2^4}+\dfrac{-2U_{001}^2U_{002}^2}{r_0^2 f_1 f_2^3}}{\left(1+\dfrac{U_{001}^2}{f_1}+\dfrac{U_{002}^2}{f_2}\right)^3}\right], \qquad 16\text{ (e)}$$

$$\phi_6(\xi,s)=(s-s_{01})^2\left[\frac{\dfrac{4U_{001}^4}{r_0^4 f_1^6}+\dfrac{-4U_{002}^4U_{002}^2}{r_1^4 f_2^6}+\dfrac{-4U_{001}^2}{r_0^4 f_1^3 f_2^3}+\dfrac{4U_{001}^2U_{002}^2}{r_0^4 f_1^3 f_2^3}}{\left(1+\dfrac{U_{001}^2}{f_1}+\dfrac{U_{002}^2}{f_2}\right)^3}\right]$$

$$+(s-s_{01})^2\left[\frac{\dfrac{4U_{001}^4}{r_0^4 f_1^6}+\dfrac{-4U_{002}^4U_{002}^2}{r_{01}^4 f_2^6}+\dfrac{-4U_{001}^2}{r_0^4 f_1^3 f_2^3}+\dfrac{4U_{001}^2U_{002}^2}{r_0^4 f_1^3 f_2^3}}{\left(1+\dfrac{U_{001}^2}{f_1}+\dfrac{U_{002}^2}{f_2}\right)^3}\right], \qquad 16\text{(f)}$$

$$\phi_7(\xi,s)=\frac{\dfrac{-2U_{001}^2}{r_0^2 f_1^3}+\dfrac{-2U_{002}^2}{r_0^2 f_2^3}}{\left(1+\dfrac{U_{001}^2}{f_1}+\dfrac{U_{002}^2}{f_2}\right)^2}$$

$$+(s-s_{01})^2\left[\frac{\dfrac{2U_{001}^2}{r_0^4 f_1^5}+\dfrac{4U_{001}^2U_{002}^2}{r_1^4 f_2^5}+\dfrac{2U_{002}^2}{r_0^4 f_2^5}+\dfrac{4U_{002}^2}{r_0^4 f_2^5}}{\left(1+\dfrac{U_{001}^2}{f_1}+\dfrac{U_{002}^2}{f_2}\right)^2}-\frac{\dfrac{4U_{001}^2}{r_0^4 f_1^6}+\dfrac{4U_{001}^2U_{002}^2}{r_1^4 f_1^3 f_2^3}+\dfrac{4U_{001}^4}{r_0^4 f_2^6}+\dfrac{4U_{001}^2U_{002}^2}{r_0^4 f_1^3 f_2^3}}{\left(1+\dfrac{U_{001}^2}{f_1}+\dfrac{U_{002}^2}{f_2}\right)^3}\right]$$

$$+(s-s_{02})^2\left[\frac{\dfrac{2U_{001}^2}{r_0^4 f_1^5}+\dfrac{4U_{001}^2U_{002}^2}{r_1^4 f_2^5}+\dfrac{2U_{002}^2}{r_0^4 f_2^5}+\dfrac{4U_{002}^2}{r_0^4 f_2^5}}{\left(1+\dfrac{U_{001}^2}{f_1}+\dfrac{U_{002}^2}{f_2}\right)^2}-\frac{\dfrac{4U_{001}^4}{r_0^4 f_1^6}+\dfrac{4U_{001}^2U_{002}^2}{r_1^4 f_2^3 f_2^3}+\dfrac{4U_{001}^4}{r_0^4 f_2^6}+\dfrac{4U_{001}^2U_{002}^2}{r_0^4 f_2^3 f_2^3}}{\left(1+\dfrac{U_{001}^2}{f_1}+\dfrac{U_{002}^2}{f_2}\right)^3}\right]. \qquad 16\text{ (g)}$$

Substituting equations (13) to (15) in equation (12) and equating coefficient of different powers of $s$, we obtain following dynamical equations:

$$\frac{\partial^2 f_1(\xi)}{\partial\xi^2}=\frac{1}{r_0^4 f_1^3(\xi)}-\frac{P_0(\alpha+\beta)}{r_0^2}\left\{\frac{\dfrac{1}{f_1^2(\xi)}+\dfrac{f_1(\xi)}{f_2^3(\xi)}}{\left(1+\dfrac{P_0}{2f_1(\xi)}+\dfrac{P_0}{2f_2(\xi)}\right)^2}\right\}$$

$$-\nu_3\left\{\frac{\dfrac{2P_0^2}{r_0^4 f_1^5(\xi)}+\dfrac{2f_1P_0^2}{r_0^4 f_2^6(\xi)}+\dfrac{4P_0^2}{r_0^4 f_1^2(\xi) f_2^3(\xi)}}{\left(1+\dfrac{P_0}{2f_1(\xi)}+\dfrac{P_0}{2f_2(\xi)}\right)^3}\right\}$$

$$+\nu_4\left\{\frac{\dfrac{6P_0}{r_0^2 f_1^4(\xi)}+\dfrac{6f_1(\xi)P_0}{r_0^4 f_2^5(\xi)}+\dfrac{P_0^2}{r_0^4 f_1^5(\xi)}+\dfrac{3P_0^2}{r_0^4 f_2^5(\xi)}+\dfrac{3P_0^2}{r_0^4 f_1^4(\xi) f_2(\xi)}+\dfrac{f_1(\xi)P_0^2}{r_0^4 f_2^6(\xi)}-\dfrac{-4P_0^2}{r_0^4 f_1^2(\xi) f_2^3(\xi)}}{\left(1+\dfrac{P_0}{2f_1(\xi)}+\dfrac{P_0}{2f_2(\xi)}\right)^3}\right\}, \qquad (17)$$

$$\frac{\partial^2 f_2(\xi)}{\partial\xi^2}=\frac{1}{r_0^4 f_1^3(\xi)}-\frac{P_0(\alpha+\beta)}{r_0^2}\left\{\frac{\dfrac{f_2(\xi)}{f_1^2(\xi)}+\dfrac{1}{f_2^2(\xi)}}{\left(1+\dfrac{P_0}{2f_1(\xi)}+\dfrac{P_0}{2f_2(\xi)}\right)^2}\right\}$$

$$-\nu_3\left\{\frac{\dfrac{2f_2(\xi)P_0^2}{r_0^4 f_1^6(\xi)}+\dfrac{2P_0^2}{r_0^4 f_2^5(\xi)}+\dfrac{4P_0^2}{r_0^2 f_1^3(\xi) f_2^3(\xi)}}{\left(1+\dfrac{P_0}{2f_1(\xi)}+\dfrac{P_0}{2f_2(\xi)}\right)^3}\right\}$$

$$+\nu_4\left\{\frac{\dfrac{6f_2(\xi)P_0}{r_0^2 f_1^5(\xi)}+\dfrac{6f_1t(\xi)P_0}{r_0^4 f_2^4(\xi)}+\dfrac{f_2(\xi)P_0^2}{r_0^4 f_1^6(\xi)}+\dfrac{3P_0^2}{r_0^4 f_1 f_2^4(\xi)}+\dfrac{3P_0^2}{r_0^4 f_1^5(\xi)}+\dfrac{P_0^2}{r_0^4 f_2^5(\xi)}-\dfrac{4P_0^2}{r_0^4 f_1^3(\xi) f_2^2(\xi)}}{\left(1+\dfrac{P_0}{2f_1(\xi)}+\dfrac{P_0}{2f_2(\xi)}\right)^3}\right\}. \tag{18}$$

$$\frac{\partial^2 s_{01}}{\partial\xi^2}=\frac{\nu P_0}{r_0^2}\left\{\frac{\dfrac{1}{f_1^3(\xi)}+\dfrac{1}{f_2^3(\xi)}}{1+\dfrac{P_0}{2f_1(\xi)}+\dfrac{P_0}{2f_2(\xi)}}\right\}$$

$$+\frac{P_0}{r_0^2}\left\{\frac{\dfrac{\nu_1}{f_1^3(\xi)}+\dfrac{\nu_1}{f_2^3(\xi)}+\dfrac{\nu_2P_0}{2f_1^4(\xi)}\dfrac{\nu_2P_0}{2f_1^3(\xi)f_2}+\dfrac{\nu_2P_0}{2f_2^4(\xi)}+\dfrac{\nu_2P_0}{2f_1(\xi)f_2^3(\xi)}}{1+(\dfrac{P_0}{2f_1(\xi)}+\dfrac{P_0}{2f_2(\xi)})^3}\right\}, \tag{19}$$

$$\frac{\partial^2 s_{02}}{\partial\xi^2}=-\frac{\nu P_0}{r_0^2}\left\{\frac{\dfrac{1}{f_1^3(\xi)}+\dfrac{1}{f_2^3(\xi)}}{1+\dfrac{P_0}{2f_1(\xi)}+\dfrac{P_0}{2f_2(\xi)}}\right\}$$

$$+-\frac{P_0}{r_0^2}\left\{\frac{\dfrac{\nu_1}{f_1^3(\xi)}+\dfrac{\nu_1}{f_2^3(\xi)}+\dfrac{\nu_2P_0}{2f_1^4(\xi)}\dfrac{\nu_2P_0}{2f_1^3(\xi)f_2}+\dfrac{\nu_2P_0}{2f_2^4(\xi)}+\dfrac{\nu_2P_0}{2f_1(\xi)f_2^3(\xi)}}{1+\dfrac{P_0}{2f_1(\xi)}+\dfrac{P_0}{2f_2(\xi)}}\right\}. \tag{20}$$

Where, $U_{001}^2=U_{002}^2=\dfrac{P_0}{2}$ is the peak power of the soliton. Equation (17) and (18) describe the evolution of the beam width of the soliton in the biased PR crystal characterized by the system parameter $\alpha,\beta,\nu_3$ and $\nu_4$. Note the absence of system parameters $\nu,\nu_1$ and $\nu_2$ in equations (17) and (18), these higher order space charge effect and diffusion phenomena do not play any role in the formation of bight spatial soliton. Equations (19) and (20) govern the dynamics of the soliton centre, which are controlled by $\nu,\nu_1$ and $\nu_2$. An important feature to note is the decoupling of equations (17) and (18) from equations (19) and (20), thus the portion of the beam center does not neither influence the formation of solitons nor their shape.

## 3. Results and discussion

In this section we first establish the existence of stationary bright solitons. In order to do that we need to solve equations (17) and (18) such that $f_1$ and $f_2$ remain constant. Therefore, we set $\dfrac{\partial^2 f_1}{\partial\xi^2}=\dfrac{\partial^2 f_2}{\partial\xi^2}=0$ in equations (17) and (18) and find out the condition for stationary soliton. The condition turns out to be:

$$r_0^2=\frac{1-\dfrac{8\nu_3P_0^2}{(1+P_0)^3}+\dfrac{4\nu_4P_0(P_0+P_3)}{(1+P_0)^3}}{\dfrac{2(\alpha+\beta)P_0}{(1+P_0)^2}},$$

where we have introduced the simplification $P_1=P_2=P_0/2$. The variation of beam radius with power for stationary solitons is shown in Fig. 1, where without any loss of generality we have assumed $f_1(\xi)=f_2(\xi)=1$. Any point which lies in any curve of Fig. 1, signifies the existence of stationary soliton with given power $P_0$. In order to examine the stability of these solitons, we have examined their behavior while they propagate in the crystal. Fig. 2 shows the variation of beam widths of two solitons while they propagate. As expected, $f_1(\xi)$ and $f_2(\xi)$ remains constant signifying stationary propagation. In Fig. 3, we have displayed variation of beam width with distance of propagation when the power of each beam is less than the threshold power required for stationary propagation. It is seen that the two soliton forming optical beams defocuses first, then focuses and again defocuses. However, the beam width always remains greater than or equal to the initial beam width. Fig. 4 depicts the variation of beam widths when the power of each beam is more than the threshold power. It is clearly evident from the figure that the normalized beam width, though oscillates, always remain less than unity. We now turn our attention to the centers of two solitons while they propagate along the crystals. Equations (17) and (18) are decoupled from equations (19) and (20), signifying the fact that the evolution of the beam centers do not depend on the formation of solitons. In Fig. 5 we have displayed movement of two beam centers inside the crystal. It is evident that their separation initially increases, then decreases and again increases. This process repeats periodically as two solitons propagate along the crystal.

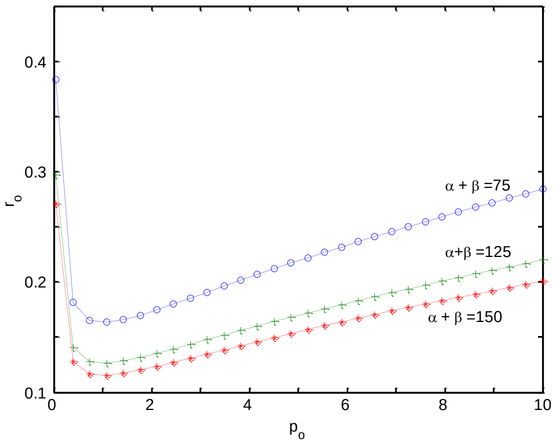


*Fig. 1. Existence curve of stationary solitons.*
$f_1(\xi=0)=f_2(\xi=0)=1$.

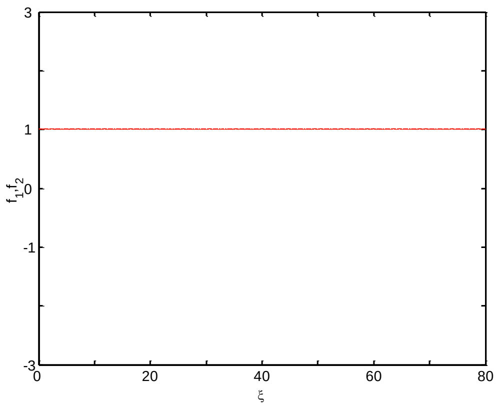


*Fig. 2. Variation of normalized beam width $f_1(\xi)$ and $f_2(\xi)$ with propagation distance of two solitons. Constant $f_1(\xi)$ and $f_2(\xi)$ signifies stationary propagation. $P_0 = 5.54$, $r_0 = 0.158$.*

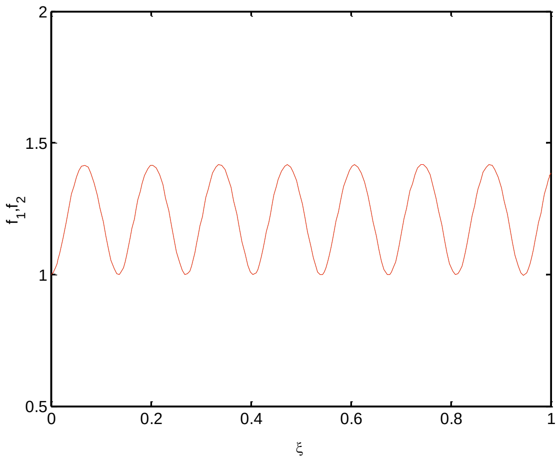


*Fig. 3. Variation of normalized beam width $f_1(\xi)$ and $f_2(\xi)$ with propagation distance when the power of each beam is less than the threshold power ($P_{0th}$) required for stationary propagation. $r_0 = 0.158$, $P_0 = 10 > P_{0th}(= 10)$.*

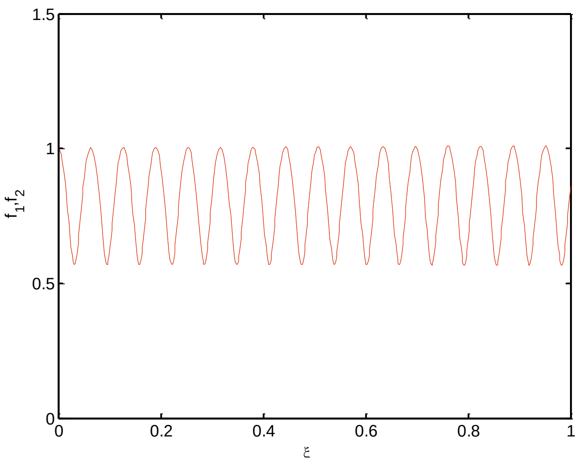


*Fig. 4. Variation of normalized beam width $f_1(\xi)$ and $f_2(\xi)$ with propagation distance when the power of each beam is more than the threshold power ($P_{0th}$) required for stationary propagation. $r_0 = 0.158$, $P_0 = 11 > P_{0th}(= 10)$.*

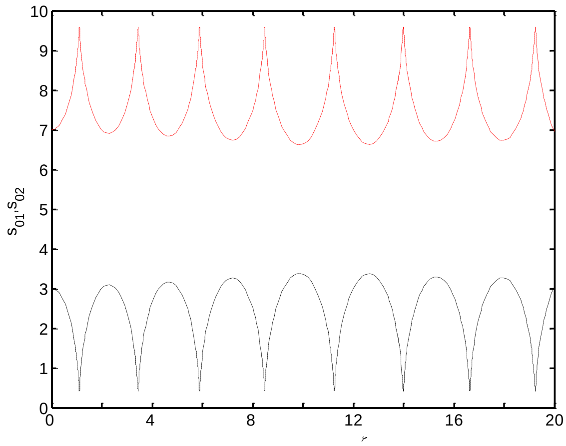


*Fig. 5. Locations of the centers of two solitons with propagation distance. $r_0 = 0.158$, $P_0 = 5.54$.*

## 4. Conclusion

We have investigated the bright screening photorefractive-photovoltaic solitons incorporating higher order space charge fields. We have found out the threshold power and beam widths for these solitons. We have plotted the existence curve of these solitons. The focusing and defocusing behavior of these solitons have been investigated. The movement of the centers of these solitons has been also investigated.

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