

The electromagnetic degree of coherence in the near field

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The mathematical estimation of the degree of correlation of optical fields in the near field has been analyzed. New approach is proposed for estimating the degree of coherence of optical waves. The possibility of transformation of spatial polarization distribution in measured intensity distribution for estimating the degree of correlation of superposing vector waves linearly polarized at the incidence plane is shown.

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1. Introduction

Coherence of electromagnetic waves, including optical ones, manifests itself in their interference resulting in spatial intensity modulation. Visibility of an interference pattern is used for estimating the degree of coherence of the waves causing such distribution [1]. On the other hand, superposition of mutually coherent but orthogonally polarized waves under paraxial approximation results in a spatial variation of the states of polarization of the resulting field; for that, any intensity modulation of the resulting field is absent.

In this paper we consider another limiting case, *viz.* superposition of waves polarized at the incidence plane, when the longitudinal component essentially contributes in the resulting field distribution, and the formalism of the Stokes parameters is not applicable. In this case, superposition of two waves results simultaneously in spatial variations of the intensity and of the polarization state, and the latter is realized at the incidence plane [2]. These effects occur because of

- transmission of radiation through optically anisotropic crystals;
- multiple light scattering of coherent radiation in turbid media, as well as transmission of optical radiation through optical waveguides; and
- heterodyning (nonlinear mixing) of optical waves of different states of polarization, as well as at the near zone of the field scattered by random phase objects.

This list of actual situations that give rise to the spatial intensity/state of polarization modulation may be extended. It is often difficult to separate the contributions caused by polarization modulation and pure interference mechanisms in the resulting spatial field distribution. In practice both kinds of modulation manifest themselves into the intensity modulation of the resulting field, because they are inseparable without the use of special arrangements.

Increasing role of such investigations is caused by the necessity of study the structure of isolated molecules as well as

obtaining of 3D images, especially in the case when a molecule is oriented along the illumination direction, *viz.* along z-axis.

The structural diagnostics of random phase objects are based on known correlations between the statistical parameters of an object and those of the scattered field [3]. The heaviest volume of information is contained in the near-field pattern, since it is not affected by spatial filtering. The optical field's polarization must be taken into account in this case [4, 5]; *i.e.*, the field has to be represented in vector form. The existence of the polarization component in the interference distribution of superposing optical fields is proved by available singularities of corresponding interference patterns [2,6]. The contribution of the longitudinal field component (LFC) then becomes important.

To describe coherence properties of optical fields we must study the interconnections among polarization and correlation parameters of optical fields.

Some papers [7, 8] propose the parameter, which characterizes the degree of mutual polarization in different points of inhomogeneously polarized optical fields. This parameter is invariant to the transformations of the coordinate system and, as a consequence, does not depend on the choice of the state of polarization of the probing beam and determines according to the following expression [7]

$$\gamma_W^2(\vec{r}_1, \vec{r}_2, \tau) = \frac{\text{tr}[W(\vec{r}_1, \vec{r}_2, \tau)W(\vec{r}_2, \vec{r}_1, -\tau)]}{\text{tr}W(\vec{r}_1, \vec{r}_1, 0)\text{tr}W(\vec{r}_2, \vec{r}_2, 0)}$$

$$= \frac{\sum_{i,j} |W_{i,j}(\vec{r}_1, \vec{r}_2, \tau)|^2}{\sum_{i,j} W_{ii}(\vec{r}_1, \vec{r}_1, 0)W_{jj}(\vec{r}_2, \vec{r}_2, 0)}, \quad (1)$$

where $W(\vec{r}_1, \vec{r}_2, \tau)$ – is the 3×3 mutual coherence matrix, defined by

$$W(\vec{r}_1, \vec{r}_2, \tau) = \langle \vec{E}_i(\vec{r}_1, \tau) \vec{E}_j^*(\vec{r}_2, \tau) \rangle, \quad (i = x, y, z) \quad (2)$$

where $i, j = x, y, z$, and we have made use of the Hermiticity relation $W_{ij}^*(\vec{r}_1, \vec{r}_2, \tau) = W_{ji}(\vec{r}_2, \vec{r}_1, -\tau)$ satisfied by the electric coherence-matrix elements. It is well-known, that the coherence properties at certain moment of time between spatiotemporal points \vec{r}_1 and \vec{r}_2 of the complex random vectors $\vec{E}(\vec{r}_1, \tau)$ and $\vec{E}(\vec{r}_2, \tau)$ can be represented by the mutual coherence matrix $W(\vec{r}_1, \vec{r}_2, \tau)$, defined by the Exp. (2). The quantity γ_W was called as the degree of coherence for electromagnetic fields [7], or the degree of mutual polarization [8]. One can note that the standard coherence matrix corresponds to the case $\vec{r}_1 = \vec{r}_2 = \vec{r}$ at the fixed moment of time. The standard coherence matrix will be called, the polarization matrix, which will thus correspond to

$$\Gamma(\vec{r}) = \langle \vec{E}(\vec{r}) \vec{E}^*(\vec{r}) \rangle \quad (3)$$

Since the degree of mutual polarization contains information about the correlation that exist between the components of the electric field at a pair of points, it might expect that there exist a connection between γ_W and the degree of polarization that characterizes correlations in a single point. It has been once more theoretically substantiated that the degree of polarization characterizes the level of disorder in each beam, *viz.* the level of decorrelation of the field components of one beam, and corresponds to the maximum of the autocorrelation function of this beam [9].

In its turn, the intrinsic coherence introduced in the paper [10] characterizes the degree of disorder between interacting beams in different points of a field and at different instants. Henceforth, for the quantitative characteristics of the intrinsic coherence we will use the term *degree of cross-correlation of the fields*, regarding these terms to be synonyms.

Really a normalized mutual coherence matrix

$$M(\vec{r}_1, \vec{r}_2, t_1, t_2) = \Gamma^{-1/2}(\vec{r}_2, t_2) W(\vec{r}_1, \vec{r}_2, t_1, t_2) \Gamma^{-1/2}(\vec{r}_1, t_1), \quad (4)$$

whose singular values $\mu_s(\vec{r}_1, \vec{r}_2, t_1, t_2), \mu_I(\vec{r}_1, \vec{r}_2, t_1, t_2)$ determine the intrinsic degrees of coherence [8, 9, 10]. From mathematical point of view, one uses the property that any matrix can be decomposed in singular values $\mu_s(\vec{r}_1, \vec{r}_2, t_1, t_2), \mu_I(\vec{r}_1, \vec{r}_2, t_1, t_2)$,

such as

$$M(\vec{r}_1, \vec{r}_2, t_1, t_2) = N_2^* D(\vec{r}_1, \vec{r}_2, t_1, t_2) N_1, \quad (5)$$

where

$$D(\vec{r}_1, \vec{r}_2, t_1, t_2) = \begin{bmatrix} \mu_s(\vec{r}_1, \vec{r}_2, t_1, t_2) & 0 \\ 0 & \mu_I(\vec{r}_1, \vec{r}_2, t_1, t_2) \end{bmatrix}, \quad (6)$$

N_1 and N_2 are unitary matrices and where $\mu_s(\vec{r}_1, \vec{r}_2, t_1, t_2) \geq \mu_I(\vec{r}_1, \vec{r}_2, t_1, t_2) \geq 0$. The matrix $D(\vec{r}_1, \vec{r}_2, t_1, t_2)$ corresponds to the normalized mutual coherence matrix of totally depolarized light described in the basis of the singular value decomposition of $M(\vec{r}_1, \vec{r}_2, t_1, t_2)$.

Let us introduce the term the degree of cross-correlation of the field. Under this term, in our approach, we understand the correlation of collinear components of the interacting beams, for example, $E_{x1}(\vec{r}_2, t_2), E_{x2}(\vec{r}_1, t_1), E_{z1}(\vec{r}_2, t_2), E_{z2}(\vec{r}_1, t_1)$, which in there turn determine the degrees of the intrinsic coherence $\mu_s(\vec{r}_1, \vec{r}_2, t_1, t_2), \mu_I(\vec{r}_1, \vec{r}_2, t_1, t_2)$, according to Exp. (4) - (6). In this sense the conceptions of the intrinsic coherence and the cross correlation are identical. We take into consideration the longitudinal field component (z -component) in order to be able to demonstrate the contribution of the polarization modulation into the degrees of intrinsic coherence. In our case of the interference of two orthogonally polarized waves the normalized matrix of mutual coherence will be put down as

$$M(\vec{r}_1, \vec{r}_2, t_1, t_2) = \begin{bmatrix} \eta_{xx}(\vec{r}_1, \vec{r}_2, t_1, t_2) & 0 \\ 0 & \eta_{zz}(\vec{r}_1, \vec{r}_2, t_1, t_2) \end{bmatrix}, \quad (7)$$

where $\eta_{i,j}(\vec{r}_1, \vec{r}_2, t_1, t_2) = \frac{\langle E_i(\vec{r}_2, t_2) E_j^*(\vec{r}_1, t_1) \rangle}{\sqrt{I_i(\vec{r}_2, t_2) I_j(\vec{r}_1, t_1)}}$, $i, j = x,$

z .

Here $\mu_s(\vec{r}_1, \vec{r}_2, t_1, t_2) = \eta_{xx}(\vec{r}_1, \vec{r}_2, t_1, t_2)$ and it describes the degree of the coherence between x -components of the two initial waves. Correspondingly $\mu_I(\vec{r}_1, \vec{r}_2, t_1, t_2) = \eta_{zz}(\vec{r}_1, \vec{r}_2, t_1, t_2)$ and describes the degree of coherence between z -components. The maximum values of the parameters μ_s and μ_I are unit and correspond to completely correlation of x - and z -components. If the both intrinsic degrees of coherence are equal to one, there exists a polarization modulation of the field that can lead to a unit modulus of the visibility of the interference pattern.

2. Statement of the problem

In the general case, the coherence must be estimated by measuring both the visibility of the resulting distribution and the deepness of modulation, *i.e.* the degree of polarization of the field. In other words, the degree of polarization and the degree of coherence of the field determined by the visibility of the interference pattern are mutually complementary measures of the cross-correlation of two fields.

Let us consider superposition of optical waves polarized at the incidence plane. Firstly, we consider superposition of two waves of equal intensities, which are linearly polarized at this plane, and whose angle of convergence equals 90° . Superposition of such waves results in homogeneous intensity distribution at the registration plane with a spatial modulation of polarization [2]. The data on the mutual coherence of the superimposing waves can be obtained by transforming the spatial distribution of polarization of the resulting field into a periodical spatial intensity distribution, by using an additional reference wave correlated at least with one of two superimposing waves and also polarized at the incidence plane. In other words, we consider the result of three-beam superposition of the waves polarized at the incidence plane $\mathbf{E}^{(1)}(\mathbf{Q}_1, t)$, $\mathbf{E}^{(2)}(\mathbf{Q}_2, t)$ and $\mathbf{E}^{(3)}(\mathbf{Q}_3, t)$; here $\mathbf{E}^{(3)}(\mathbf{Q}_3, t)$ is the reference wave. $\mathbf{Q}_1, \mathbf{Q}_2, \mathbf{Q}_3$ - are the coordinates of the pinholes sources. A point \mathbf{r} determines the analyze point in the registration plane.

Thus, in the general case, at the time t a random electromagnetic field, formed by the sources $\mathbf{Q}_1, \mathbf{Q}_2, \mathbf{Q}_3$ and displayed at the point \mathbf{r} on the screen can be put down as,

$$\mathbf{E}(\mathbf{r}, t) = \mathbf{E}(\mathbf{Q}_1, t) \frac{\exp(ikR_1)}{R_1} + \mathbf{E}(\mathbf{Q}_2, t) \frac{\exp(ikR_2)}{R_2} + \mathbf{E}(\mathbf{Q}_3, t) \frac{\exp(ikR_3)}{R_3}, \quad (8)$$

$$V^\varphi = 2 \sum_{ij} \frac{\sqrt{\text{tr}[W(\mathbf{Q}_1, \mathbf{Q}_1, 0)] \text{tr}[W(\mathbf{Q}_2, \mathbf{Q}_2, 0)]}}{\varphi_{ij}^{(1)}(\mathbf{r}) + \varphi_{ij}^{(2)}(\mathbf{r}) + \varphi_{ij}^{(3)}(\mathbf{r})} \left| \eta_{ij}^{(1,2)} \right| + 2 \sum_{ij} \frac{\sqrt{\text{tr}[W(\mathbf{Q}_1, \mathbf{Q}_1, 0)] \text{tr}[W(\mathbf{Q}_3, \mathbf{Q}_3, 0)]}}{\varphi_{ij}^{(1)}(\mathbf{r}) + \varphi_{ij}^{(2)}(\mathbf{r}) + \varphi_{ij}^{(3)}(\mathbf{r})} \left| \eta_{ij}^{(1,3)} \right| \cos[\varphi_1] + 2 \sum_{ij} \frac{\sqrt{\text{tr}[W(\mathbf{Q}_2, \mathbf{Q}_2, 0)] \text{tr}[W(\mathbf{Q}_3, \mathbf{Q}_3, 0)]}}{\varphi_{ij}^{(1)}(\mathbf{r}) + \varphi_{ij}^{(2)}(\mathbf{r}) + \varphi_{ij}^{(3)}(\mathbf{r})} \left| \eta_{ij}^{(2,3)} \right| \cos[\varphi_2], \quad (10)$$

where φ_1 and φ_2 - the phase differences of two initial superposing waves and the reference wave. The visibility modulation depth (VMD) is determined as

$$M = \max[V^\varphi] - \min[V^\varphi] = 2 \sum_{m, ij} \frac{\sqrt{\text{tr}[W(\mathbf{Q}_m, \mathbf{Q}_m, 0)] \text{tr}[W(\mathbf{Q}_3, \mathbf{Q}_3, 0)]}}{\varphi_{ij}^{(m)}(\mathbf{r}) + \varphi_{ij}^{(3)}(\mathbf{r})} \left| \eta_{ij}^{(m,3)} \right| \quad m = 1, 2; i, j = x, z \quad (11)$$

Choosing the reference wave to be completely correlated with one of the initial waves, to say $\left| \eta^{(1,3)} \right| = 1$, one can see that the VMD of an interference pattern M characterizes, up to the constant depending on the intensity ratio, the degree of mutual coherence of the reference wave and the second of the initial waves, i.e. $M = K \cdot \left| \eta^{(2,3)} \right|$.

here k - is the wavenumber, $R_1 = |\mathbf{r} - \mathbf{Q}_1|$, $R_2 = |\mathbf{r} - \mathbf{Q}_2|$, $R_3 = |\mathbf{r} - \mathbf{Q}_3|$ are distances of point \mathbf{r} from the pinholes sources.

The resulting time-averaged intensity distribution at the observation plane [12, 13] gives the possibility to estimate the visibility of the resulting interference pattern. Using the conventional definition of the visibility, one can write:

$$V = \frac{\max[I(\mathbf{r})] - \min[I(\mathbf{r})]}{\max[I(\mathbf{r})] + \min[I(\mathbf{r})]} = \sum_{m, n} \sum_{ij} \frac{\sqrt{\text{tr}[W(\mathbf{Q}_m, \mathbf{Q}_m, 0)] \text{tr}[W(\mathbf{Q}_n, \mathbf{Q}_n, 0)]}}{\varphi_{ij}^{(1)}(\mathbf{r}) + \varphi_{ij}^{(2)}(\mathbf{r}) + \varphi_{ij}^{(3)}(\mathbf{r})} \left| \eta_{ij}^{(m,n)} \right|, \quad (9)$$

where $m \neq n$, $m, n = 1, 2, 3$, $i, j = x, z$.

It follows from Eq. (6) that the visibility of an interference pattern allows to determine, within to a constant, the degree of mutual coherence of the initial superposing waves, $\left| \eta^{(1,2)} \right|$. Changing the phase of the reference wave within the interval $0..2\pi$ results in periodical changing of visibility of the registered interference pattern following the harmonic law:

Accounting that $\left| \eta^{(1,3)} \right| = 1$, one concludes that $\left| \eta^{(2,3)} \right| = \left| \eta^{(1,2)} \right|$. Thus, by proper choice of intensities of the interfering waves one can obtain $K \rightarrow 1$, and $\left| \eta^{(1,2)} \right|$ will be determined by the VMD of the interference pattern: $M = \left| \eta^{(1,2)} \right|$.

3. Results and discussion

It is possible to perform a correct experiment for estimating of superposition of orthogonally polarized waves in the incidence plane if some factors are taken into account. To avoid distortions introduced by the optical system, we must take into account the fact that the propagation of radiation through a microscope is accompanied by the change of a cone angle of the beams, so that this angle differs from the right one. This leads to the violation of the strict orthogonality of the electrical vectors of the interfering beams and manifests itself in the

spatial intensity modulation. The initial attempt of holographic estimation of the degree of coherence of such waves has been described in paper [14], where for providing “purity” of an experiment an immersion liquid is used at the arrangement shown in Fig. 1, so that the angle of convergence of plane superposing waves is 90° , and the only spatial polarization modulation of the resulting distribution takes place. Such a system fulfills the strict angular requirements for the waves in the recording region.

On the other hand, usage of an immersion provides effective reconstruction of a beam at the readout stage. The use of a holographic technique is caused by the necessity of registration of spatial intensity distribution at very high frequencies.

It is clear accounting the results of this study, that holographic registration of the resulting interference distribution would be carried out step-by-step, changing a phase of the reference wave with interval 0.2π for revealing the maximal modulation depth of this distribution.

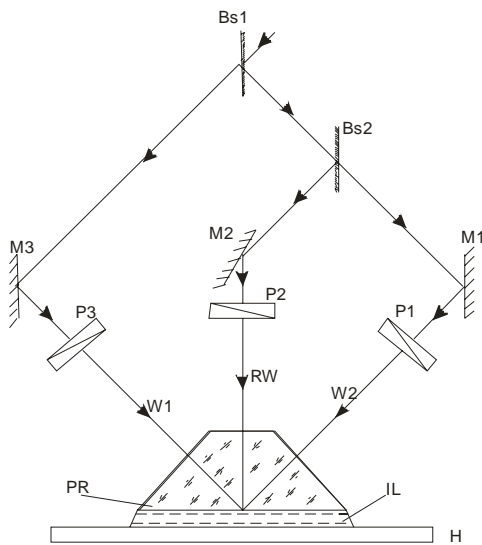


Fig. 1. Optical arrangement for holographic experiment: Bs1 and Bs2, beam splitters; M1, M2, and M3, mirrors; P1, P2, and P3, polarizers; PR, prism; IL, immersion liquid; H, hologram.

Fig. 2 illustrates the polarization modulation scheme for the case when two completely coherent plane waves linearly polarized at the incidence plane with the angle of convergence 90° . As it has been mentioned above, using the coherent reference wave, which is linearly polarized at the incidence plane and normally incident at the registration plane, enables to visualize the polarization distribution. For that, visibility of an interference pattern depends on the phase of the reference wave. So, if intensity of this wave exceeds intensity of the each initial waves, one registers modulation of visibility of the resulting pattern within the interval from zero to unity. This is confirmed by the results of computer simulation

shown in Fig. 3. This fact just explains why the experiments on detection of polarization modulation using the holographic technique in papers [2, 14] are not always successful. At the same time, the VMD equal unity strictly corresponds to the degree of mutual coherence of the initial waves and serves as the criteria for diagnostics of this parameter, Fig. 3, curve 1.

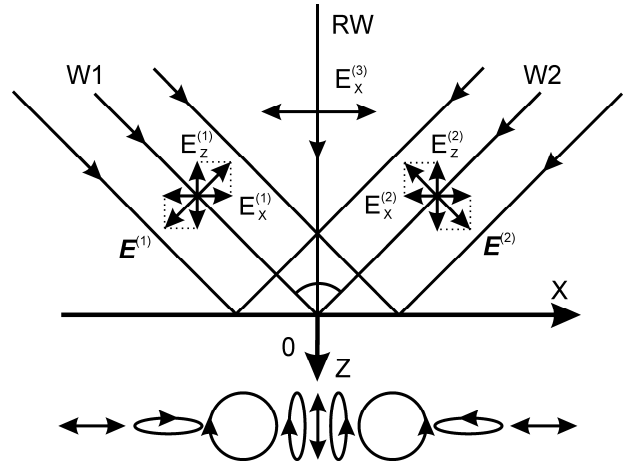


Fig. 2. the upper part - polarization modulation scheme (W1, W2 are obliquely incident waves, RW is a reference wave); the bottom - illustration of the spatial polarization modulation of the resulting field from linear polarization state through elliptical and circular and finally to linear polarization state.

Superposition of plane waves of equal intensities linearly polarized at the incidence plane whose degree of mutual coherence equals zero at the same registration scheme also results in homogeneous intensity distribution at the registration plane. The use of the plane reference wave coherent with one of the initial waves enables to visualize the intensity distribution with the visibility at the level 0.5 for intensity of the reference wave exceeding intensities of the initial waves. Of course, in this case changing the phase of the reference wave does not result in modulation of visibility of the resulting pattern, Fig. 3, curve 2. It means that the VMD equals zero that is in quite correspondence with the degree of mutual coherence of the initial superimposing waves. The experiments carried out for the cases when $0 < |\eta^{(1,2)}| \leq 1$ completely proved the conclusion that the VMD of a pattern corresponds to the magnitude $|\eta^{(1,2)}|$ of the superimposing waves. The results of computer simulation for the same arrangement and the same states of polarization of the superposing initial and reference waves but for different magnitudes of the degree of mutual coherence of the initial waves show that the VMD of an interference pattern strictly corresponds to the degree of mutual coherence of these waves. These results are illustrated in Fig.3, curves 3, 4, 5.

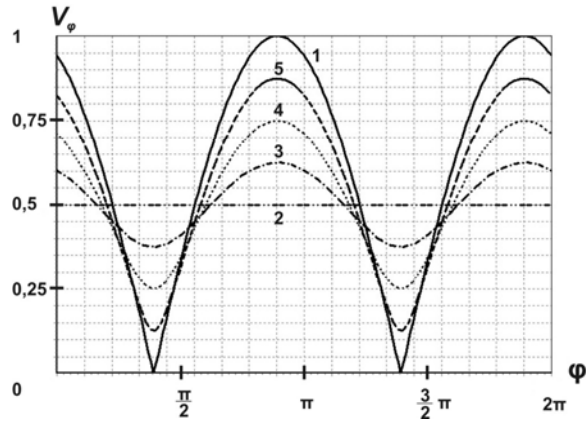


Fig. 3. Dependences of visibilities V of the interference patterns resulting from three-beam superposition on phase of the reference wave φ : curve 1 - for the case of complete coherent waves $|\eta^{(1,2)}|=1$, the VMD corresponds to $M = 1$; curve 2 - $|\eta^{(1,2)}|=0$, the VMD corresponds to $M = 0$; curve 3 - $|\eta^{(1,2)}|=0.25$, the VMD corresponds to $M = 0.25$; curve 4 - $|\eta^{(1,2)}|=0.5$, the VMD corresponds to $M = 0.5$; curve 5 - $|\eta^{(1,2)}|=0.75$, the VMD corresponds to $M = 0.75$.

4. Conclusions

Thus, the contribution of the polarization component in the correlation of the vector optical fields has been substantiated and experimentally illustrated. The feasibilities for using the data contained in the structure and peculiarities of the spatial polarization modulation of a field resulting from superposition of vector optical fields for estimation of the degree of correlation of these fields. The algorithm for estimation of the degree of coherence of vector optical waves consists in searching for optimal

means of transformation of polarization distributions of a field into measurable intensity distributions providing necessary data.

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