

The transformation of the polyphase synchronous machine (N phases) to the whole of fictitious machines mono and diphas mechanically coupled and magnetically uncoupled

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In this article, we will simulate the synchronous polyphase machine at permanent magnets, with a load, for using the mathematical model associated with the multi-machine concept. The simulation allows us to see the machine behavior and check if the selected model can be used to control the speed and torque by transforming the actual machine into a set of fictitious machines. The polyphase machines are developed mainly in the field of variable speed drives of high power because increasing the number of phases on the one hand allows to reduce the dimensions of the components in power modulators energy and secondly to improve the operating safety. By a vector approach (vector space), it is possible to find a set of single-phase machine and / or two-phase fictitious, equivalent to synchronous polyphase machine. These fictitious machines are coupled electrically and mechanically but decoupled magnetically. This approach leads to introduce the concept of the equivalent machine (multimachine multiconverter system MMS) which aims to analyze systems composed of multiple machines (or multiple converters) in electric drives. A first classification multimachine multiconverter system follows naturally from MMS formalism. We present an example of a pentaphase synchronous machine.

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1. Introduction

Through the many advances in technology, the power applications high and average at speed variable are increasingly made on the basis of the whole electrical machinery-static converters. For applications of high power density, low rotor losses and reduced inertia, the synchronous machines with permanent magnets [1] are best suited. However with the traditional structures of static converters and high power machines, the power transmitted between the power source and the mechanically receiver cannot be treated appropriately. The use of current switches associated with machine double-star [2-4] on the one hand allows to reduce the power transmitted by each converter and, secondly, to reduce the torque ripple of the machine.

Despite this improvement, the torque ripples are important, especially for low speeds. Despite this improvement, the torque ripples are important, especially for low speeds. The polyphase machines are an interesting alternative to reducing constraints applied to the switches and coils.

Indeed, the increase in the number of phases allows a fractionation of power, and therefore a reduction in switched voltages at a given current. In addition, these machines can reduce the [5] amplitude and increasing the frequency of the torque ripple, which allows at the

mechanical loading of filter them more easily. Finally, increasing the number of phases provides increased reliability by allowing run, one or more faulted phases [2,6]. The polyphase machines are found in areas such as marine, railway, petrochemical industry, avionics, automotive, etc.

In our research we try to give a simple mathematical model based on the multimachine concept that will allow us to study the behavior of the polyphase machine (pentaphase) by simulation Matlab Simulink.

2. Principle of vectorial modelling

2.1 Presentation of assumptions

- The effects of skin, shock absorbers, saturation, and variation of reluctance of the magnetic circuit are neglected.

- The e.m.f. induced in the stator windings are solely due to the rotor magnets which have a shape that is due only to the magnets and the structure of the windings. Armature reaction magnetic (due to stator currents) does not change the form of the e.m.f..

-The phases are the same and shifted by an angle:

$$\alpha = \frac{2\pi}{n}$$

n : is the number of phase of the machine.

-The e.m.f. induced depends only on the speed of the rotor and structural parameters of the machine such as:

The figure shows a bipolar machine where g_k is a voltage, current or flux on the k phase denoted g_k .

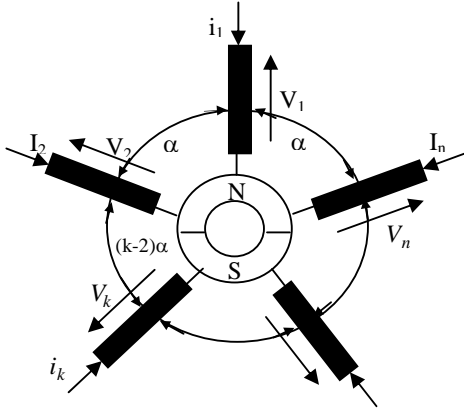


Fig. 1. The polyphase synchronous machine.

2.2 Definition of a natural base

If we associate the n -phase machines [7, 5] Euclidean vector space E^n of dimension n , an orthonormale basis of the space B^n :

$$B^n = \{\vec{x}_1^n, \vec{x}_2^n, \dots, \vec{x}_n^n\}$$

It is called natural when the vector g can be written as:

$$\vec{g} = g_1 \cdot \vec{x}_1^n + g_2 \cdot \vec{x}_2^n + \dots + g_n \cdot \vec{x}_n^n \quad (1)$$

g_1, g_2, \dots, g_n : Measurable magnitudes of the stator phases. Consequently in this space can therefore be defined vectors:

- Voltage:

$$\vec{v} = v_1 \cdot \vec{x}_1^n + v_2 \cdot \vec{x}_2^n + \dots + v_n \cdot \vec{x}_n^n$$

- Current:

$$\vec{i} = i_1 \cdot \vec{x}_1^n + i_2 \cdot \vec{x}_2^n + \dots + i_n \cdot \vec{x}_n^n$$

The voltage vector of the machine is:

$$\vec{v} = R_s \cdot \dot{\vec{i}} + \left[\frac{d\vec{\Phi}_s}{dt} \right]_{/B^n} + \vec{e} \quad (2)$$

The projection of the machine voltage v to a vector k of the voltage of a phase stator gives:

$$v_k = \vec{v} \cdot \vec{x}_k^n = R_s \cdot \dot{i}_k + \left[\frac{d\vec{\Phi}_{sk}}{dt} \right]_{/B^n} + e_k \quad (3)$$

This is the equation of the stator voltage for a phase:

- $\vec{\Phi}_{sk}$: the flux in phase k created by the stator currents.
- e_k : is the e.m.f. induced in the phase k created by rotor magnets. Assumptions of unsaturation and no reluctance variation can define a linear relationship $\vec{\Phi}_s = \lambda(\vec{i})$ between the current vector and the stator flux more usually written in the form of a matrix with constant coefficients:

$$\vec{\Phi}_s = [L_s^n] * [i_n]$$

$$[L_s^n] = \begin{pmatrix} L_{s_1s_1} & L_{s_1s_2} & \dots & L_{s_1s_n} \\ L_{s_2s_1} & L_{s_2s_2} & \dots & L_{s_2s_n} \\ \vdots & \vdots & \ddots & \vdots \\ L_{s_ns_1} & L_{s_ns_2} & \dots & L_{s_ns_n} \end{pmatrix} \quad (4)$$

$L_{s_k s_k}$ Is the inductance of a stator phase

$L_{s_j s_k}$ Mutual inductance between stator phases.

The instantaneous power is transiting in the machine:

$$p = \sum_{k=1}^n v_k \cdot i_k = \vec{v} \cdot \vec{i} \quad (5)$$

By replacing the expression vector voltage (2), we obtain the following equation:

$$p = R_s (\dot{\vec{i}})^2 + \left[\frac{d\vec{\Phi}_s}{dt} \right]_{/B^n} \cdot \dot{\vec{i}} + \vec{e} \cdot \dot{\vec{i}} \quad (6)$$

- The electrical power lost by Joule effect is:

$$p_j = R_s \dot{\vec{i}}^2$$

- The magnetic power is:

$$p_w = \left[\frac{d\vec{\Phi}_s}{dt} \right]_{/B^n} \cdot \dot{\vec{i}}$$

- Electromagnetic power is:

$$p_{em} = \vec{e} \cdot \dot{\vec{i}}$$

- The electromagnetic torque is:

$$C_{em} = \frac{\vec{e} \cdot \dot{\vec{i}}}{\Omega}$$

Ω is the instantaneous speed of the rotor.

2.3 Modeling of the machine N-phased in a base ensuring a magnetic decoupling

The relation $\vec{\Phi}_s = \lambda(\dot{\vec{i}})$, which is one of the [8, 9] morphisme, between the current vector and stator flux remains true whatever the base of the space E^n chosen [10].

The base where exist the magnetic decoupling is that in which one coordinated stator flux vector can be expressed as a function of a single coordinate of the current vector (matrix diagonal inductance).

Diagonalization of an inductance matrix requires research of the eigenvalues and eigenvectors associated with them. We define the eigenvalues Λ_k the morphisme λ as being solutions of the characteristic equation:

$$\det(\Lambda[I_n] - [L_s^n]) = 0$$

$[I_n]$ is the identity matrix of dimension n.

The eigenvalues are real because the inductance matrix is symmetric. The hypothesis of regularity spatial of phases construction, allows us to affirm that inductance matrix is circulant. Circularity property allows us to calculate analytically the eigenvalues by using the formula for circulant determinant. These two conditions are respected; the complex eigenvalues are given by the solutions of the equation:

$$\prod_{l=1}^n \left(\Lambda - \sum_{k=1}^n \left(L_{s_1 s_k} \cdot e^{\frac{2j\pi(l-1)(k-1)}{n}} \right) \right) = 0 \quad (7)$$

j is the complex operator.

Equation (7) is divided into n equations each having a specific value as a solution of the morphisme λ . These n eigenvalues are given in complex forms which are associated eigenvectors.

These complex coordinate vectors form an orthonormale basis of the Hermitian space associated to machine. We want, as with the transform Concordia, work with real coordinates eigenvectors associated with real eigenvalue. The inductance matrix is symmetrical, therefore the values Λ_k are real. We notice that:

$$\Lambda_k = \Lambda_{n-k+2}$$

Then there exists an eigenvector associated with the eigenvalue Λ_k .

It is therefore in the plane spanned by the vectors of an infinite orthonormale bases generated by the eigenvectors.

The property $e^{j(n-k)2\pi/n} = e^{-jk2\pi/n}$ allows determining an orthonormale basis composed of eigenvectors with real coefficients such that:

The new matrix inductance $[L_s^d]$, characteristic morphisme in the new basis $B^d = \{\vec{x}_1^d, \vec{x}_2^d, \dots, \vec{x}_n^d\}$ becomes:

$$[L_s^d] = \begin{pmatrix} \Lambda_1 & 0 & \dots & 0 \\ 0 & \Lambda_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \Lambda_n \end{pmatrix} \quad (8)$$

This matrix is diagonal and we recall that the inductances of the matrix are at least equal in pairs (eigenvalue of multiplicity of order 2).

This research eigenvalues associated to the inductance matrix can formulate a generalized Concordia transformation with transition matrix as the natural base to base decoupling [11].

Bases of departure and arrival are orthonormale,

This transformation has the property to preserve the instantaneous power regardless of the base in which it is expressed.

2.4 Equations of the machine in a base of decoupling

A vector \vec{g} the initial space decomposes into:

$$\vec{g} = \sum_{g=1}^{g=N} \vec{g}_g \quad (9)$$

Let N be subspaces each associated to an eigenvalue Λ_g . \vec{g}_g is the projection of the vector \vec{g} on the subspace E^g .

The new equation of the flux vector and current:

$$\vec{\Phi}_s = \sum_{g=1}^{g=N} \vec{\Phi}_{sg} = \sum_{g=1}^{g=N} \Lambda_g \vec{i}_g \quad (10)$$

Allows writing in each sub-space, a new voltage equation:

$$\vec{v}_g = R_s \vec{i}_g + \left[\frac{d\vec{\Phi}_{sg}}{dt} \right]_{/E^n} + \vec{e}_g \quad (11)$$

Using the property (10) into (11)

$$\vec{v}_g = R_s \vec{i}_g + \Lambda_g \left[\frac{d\vec{i}_g}{dt} \right]_{/E^g} + \vec{e}_g \quad (12)$$

The electrical power which transits into the real machine is expressed by [12]:

$$p = \vec{v} \cdot \dot{\vec{i}} = \sum_{g=1}^N \vec{v}_g \cdot \dot{\vec{i}}_g \quad (13)$$

By replacing the expression of voltage (12) in the power equation (13) we obtain:

$$p = \sum_{g=1}^N \left(R_s (\dot{\vec{i}}_g)^2 + \Lambda_g \left[\frac{d\dot{\vec{i}}_g}{dt} \right]_{/E^g} \cdot \dot{\vec{i}}_g + \vec{e}_g \cdot \dot{\vec{i}}_g \right) \quad (14)$$

The previous equation shows that the energy transits through N fictitious machines, independent magnetically associated with N eigenspaces.

Consequently, the actual torque is written:

$$C = \sum_{g=1}^N C_g \quad (15)$$

With:

$$C_g \cdot \Omega = \vec{e}_g \cdot \dot{\vec{i}}_g.$$

Remark:

Equation (14) shows that each fictive machine produces a torque participate in the creation of a total torque. These N fictitious machines are mechanically coupled: they rotate at the same speed and are rigidly coupled to the same mechanical shaft.

3. Application to pentaphase machine

3.1 Presentation of the machine

We model [13, 14], for the application, a pentaphase synchronous machine with permanent magnets. This machine is represented symbolically in Fig. 2

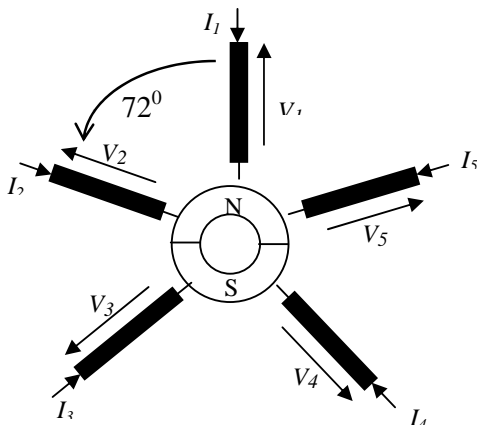


Fig. 2. The pentaphase machine.

3.2 Modeling of the machine in the natural basis

We associate the five phases a Euclidean vector space E^5 of dimension 5.

Let us write in an orthonormale base B^n the equation in tension of the machine:

$$\vec{v} = R_s \cdot \dot{\vec{i}} + \left[\frac{d\vec{\Phi}_s}{dt} \right]_{/B^n} + \vec{e} \quad (16)$$

The Orthonormale base:

$$B^n = \{ \vec{x}_1, \vec{x}_2, \vec{x}_3, \vec{x}_4, \vec{x}_5 \}$$

$$\vec{\Phi}_s = \lambda(\dot{\vec{i}}) \quad (17)$$

$$[L_s^n] = \begin{pmatrix} L & M_1 & M_2 & M_2 & M_1 \\ M_1 & L & M_1 & M_2 & M_2 \\ M_2 & M_1 & L & M_1 & M_2 \\ M_2 & M_2 & M_1 & L & M_1 \\ M_1 & M_2 & M_2 & M_1 & L \end{pmatrix} \quad (18)$$

With:

- L the inductance of a phase ($L = L_p + l_f$);
- M_1 Mutual inductance between two phases shifted of $\pm \frac{2\pi}{5}$;
- M_2 Mutual inductance between two phases shifted of $\pm \frac{4\pi}{5}$.

The vector e.m.f.:

$$\vec{e} = e_1 \cdot \vec{x}_1 + e_2 \cdot \vec{x}_2 + e_3 \cdot \vec{x}_3 + e_4 \cdot \vec{x}_4 + e_5 \cdot \vec{x}_5$$

With

$$e_k = \sum_{h=1}^{\infty} E^h \cdot \sin \left(h \left(p\theta - (k-1) \frac{2\pi}{5} \right) \right)$$

$E^h = k_{fem}^h \cdot \Omega$ Is the maximum value of the harmonic of e.m.f. of row H with k_{fem}^h the coefficient of e.m.f. and Ω number of revolutions of the rotor.

3.3 Modeling in a base of decoupling

There is an orthonormale [14] base $B^d = \{ \vec{x}_z, \vec{x}_{p\alpha}, \vec{x}_{p\beta}, \vec{x}_{s\alpha}, \vec{x}_{s\beta} \}$

In which the matrix inductance is diagonal:

$$[L_s^d] = \begin{pmatrix} \Lambda_1 & 0 & 0 & 0 & 0 \\ 0 & \Lambda_2 & 0 & 0 & 0 \\ 0 & 0 & \Lambda_5 & 0 & 0 \\ 0 & 0 & 0 & \Lambda_3 & 0 \\ 0 & 0 & 0 & 0 & \Lambda_4 \end{pmatrix} \quad (19)$$

It appears double values then:

$$\begin{aligned} \Lambda_1 &= L + 2(M_1 + M_2) \\ \Lambda_2 &= \Lambda_5 = L - 2 \cdot \left(M_1 \cdot \cos\left(\frac{3\pi}{5}\right) + M_2 \cdot \cos\left(\frac{\pi}{5}\right) \right) \\ \Lambda_3 &= \Lambda_4 = L - 2 \cdot \left(M_1 \cdot \cos\left(\frac{\pi}{5}\right) + M_2 \cdot \cos\left(\frac{3\pi}{5}\right) \right) \end{aligned}$$

Inductances are associated the clean vectors.

There is a single eigenvalue and two eigenvalues double. This property enables us to break up the vector space E^5 into three orthogonal subspaces with knowing:

- A subspace E^z generated by the clean vector (\vec{x}_z) associated the eigenvalue $\Lambda_z = \Lambda_1$.

This subspace is a line called homopolar;

- A subspace E^p generated by the clean vectors $(\vec{x}_{p\alpha}, \vec{x}_{p\beta})$ associated the eigenvalue:

$$\Lambda_p = \Lambda_2 = \Lambda_5$$

This subspace is a plan called the main thing.

- A subspace E^s generated by the clean vectors $(\vec{x}_{s\alpha}, \vec{x}_{s\beta})$ associated the

eigenvalue $\Lambda_s = \Lambda_3 = \Lambda_4$. This subspace is a plan called secondary.

The vector \vec{g} is:

$$\vec{g} = \vec{g}_z + \vec{g}_p + \vec{g}_s$$

3.4 Equivalence between real and fictitious machine

A fictitious machine [9] can be associated each subspace, respectively

- A diphas machine associated the principal plan having the electric time-constant and e.m.f. induced the most significant;

- A diphas machine associated the secondary plan having the electric time-constant weaker and e.m.f. less significant

The fictitious homopolar machines are magnetically decoupled and depend of his own current homopolar.

3.5 Simulation of the pentaphase machine

We will implement the model of the machine on the numerical simulation software through its Matlab Simulink module and considering that the first harmonic. We simulate the machine with the load torque $C_r=2$ N/m

After simulation we get e.m.f. and currents are sinusoidal (Fig. 3) and (Fig. 4):

$$e_k = E_{\max} \sin(\omega t - \frac{2(k-1)\pi}{5}), k = 1, \dots, 5$$

$$i_k = I_{\max} \sin(\omega t - \frac{2(k-1)\pi}{5}), k = 1, \dots, 5$$

K indicates the number of the phase, for our machine it y' has 5 phases (k phases).

e.m.f. are in phase with the currents:

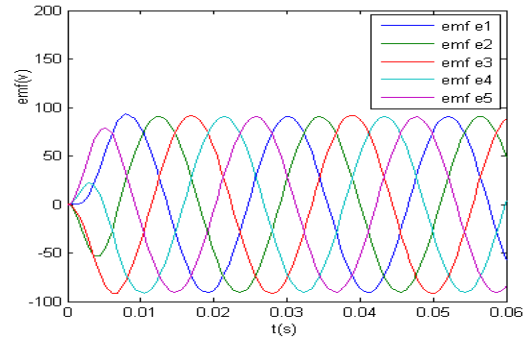


Fig. 3. e.m.f. of the pentaphase machine.

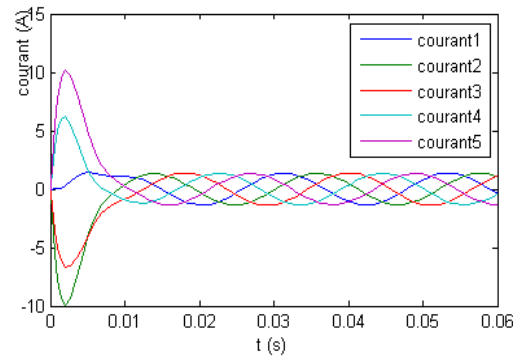


Fig. 4. Current of the pentaphase machine.

Interpretation (Figs. 5et 6): we note that the curves of currents and e.m.f are sinusoidal after a brief transitional regime.

3.5.1. e.m.f. of the fictitious machines in the base of Concordia:

-e.m.f. in the principal machine:

$$e_{\alpha p} = \sqrt{5/2} E_{\max} \sin(\omega t)$$

$$e_{\beta p} = -\sqrt{5/2} E_{\max} \cos(\omega t)$$

-e.m.f. in the secondary machine:

$$e_{\alpha s} = 0$$

$$e_{\beta s} = 0$$

e.m.f. in the homopolar machine

$$e_z = 0$$

We find although in this case only the principal machine can provide the couple

3.5.2. Currents in the fictitious machines in the base of Concordia:

-The current in the principal machine:

$$i_{\alpha p} = \sqrt{5/2} I_{\max} \sin(\omega t)$$

$$i_{\beta p} = -\sqrt{5/2} I_{\max} \cos(\omega t)$$

-The current in the secondary machine:

$$i_{\alpha s} = 0 \text{ And } i_{\beta s} = 0$$

-The current in the homopolar machine:

$$e_z = 0$$

3.5.3. e.m.f. in the fictitious machines in the base of Park (fig. 5)

$$e_{pd} = 0, e_{sd} = 0 \text{ and } e_{sq} = 0$$

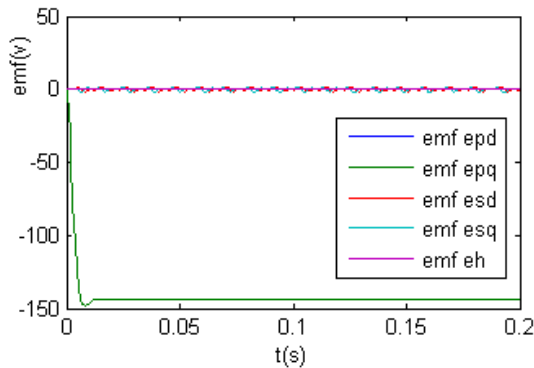


Fig. 5. e.m.f. of the fictitious machines in the base of Park.

Interpretation (Fig. 5): We see in the base of the park, the components of e.m.f. are zero at except the e.m.f. principal quadratic which is a constant.

3.5.4. Currents in the fictitious machines in the base of Park (Fig. 6)

$$i_{sd} = 0 \text{ } i_{sq} = 0$$

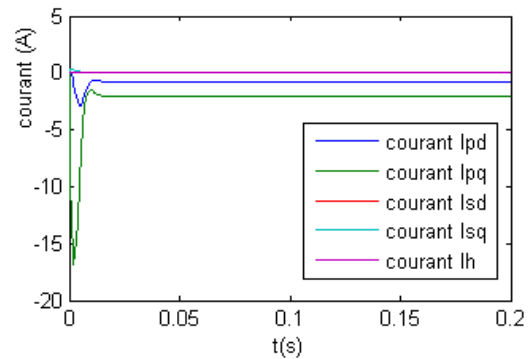


Fig. 6. Current of the fictitious machines in the base of Park.

Interpretation (Fig. 6): We notice in the basis of Park, the currents of the homopolar machine and secondary, are zero but the currents of the principal machine are constants nonzero.

3.5.5. Electromagnetic torque

$$C_n = \frac{1}{\Omega} (e_z i_z + \overrightarrow{e_{\alpha\beta p}} i_{\alpha\beta p} + \overrightarrow{e_{\alpha\beta s}} i_{\alpha\beta s})$$

$$C_n = \frac{1}{\Omega} (e_{pq} i_{pq}) = \frac{5}{2} \frac{E_{\max} I_{\max}}{\Omega}$$

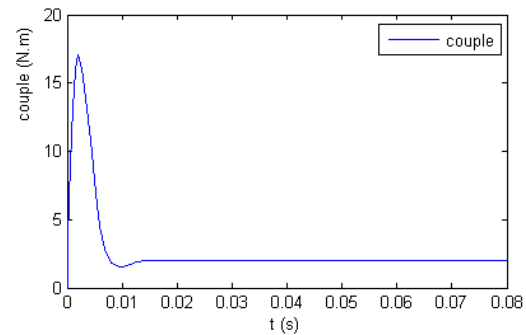


Fig. 7. Couples electromagnetic.

Interpretation (Fig. 7), we remark that the torque is constant and is equal to the load torque.

4. Interpretation of simulation

The simulations allow us to observe the e.m.f. in fictitious machines and currents in the basis of Park. The transformation of Park applied to these magnitudes indicates that only the component (q) of the e.m.f. of the principal machine is a nonzero constant (Fig. 5). We observe that the homopolar e.m.f. is zero, which is coherent for a machine sinusoidal, balanced. Accordingly, the homopolar machine produces no torque ($C_h = 0$) as well as for the secondary machine ($C_s = 0$). So the principal machine (diphase) alone produces torque. We find that the torque is constant

$$C = C_h + C_p + C_s = C_p$$

With:

$$C_h = 0 \text{ et } C_s = 0$$

5. Conclusion

A polyphase machine is made up of N windings out of phase spatially of $2\pi/N$ and supplied with tensions out of phase temporally of $2\pi/N$. These machines are characterized by a magnetic coupling between phases.

The generalization of the method of the vector of space makes it possible to define a basic change of dimension N, implying a simplification of the study of the machine by the diagonalization of the matrix inductance.

This basic change led to under orthogonal vector spaces of dimension 2 or 1.

Each subspace can thus be independent. The concept of fictitious machine of dimension 1 or 2 is then introduced.

One transforms a magnetic coupling constraining and difficult to a coupling simple.

Association vectorial modeling concept multimachines makes it possible to regard a polyphase machine as equivalent to a whole of mono and/or two-phase machines fictitious mechanically coupled.

The study of machine complexes is thus summarized in several studies of simple machines. Therefore the selected model can be used for the polyphase machine control.

References

- [1] E. Semail, Thèse de doctorat, Université des Sciences et Technologies de Lille (USTL), juin (2000).
- [2] D. Hadiouche, thèse de doctorat de l'UHP, Nancy 1, décembre (2001).
- [3] N. Moubayed, F. Meibody-Tabar, B. Davat, Electrimacs'99 (conference international), Portugal (1999).
- [4] D. Hadiouche, H. Razik, A. Rezzoug, ICEM (2000), August Espoo (Finland), pp 392-396.
- [5] E. Semail, Thèse de doctorat, USTL, Juin (2000).
- [6] F. Menneron, M. F. Benkhoris, A. Guéraud. Modélisation et simulation d'une machine asynchrone double étoile alimentée en tension pour la propulsion par POD. La conversion électromécanique directe. 4 février (1999)-ENS Cachan SEE.
- [7] J. P. Martin, S. Pierfederici, F. Meibody-Tabar, B. Davat, Etude de deux structures segmentées d'alimentation en tension d'une MSAP. Comparaison de leurs performances en mode normal et en mode dégradé. Séminaire SMM 2000, GDR-SDSE, Lille, 5-6 Février (2001).
- [8] E. Semail, A. Bouscayrol, J. P. Hautier. EPJ AP, **22**(3), 207 (2003).
- [9] J-P. Martin, E. Semail, S. Pierfederici, A. Bouscayrol, F. Meibody-Tabar et B. Davat Commande d'une machine polyphasée basée sur la représentation vectorielle généralisée. Séminaire SMM 2002, GDR SDSE, 25-26 octobre (2001), Cachan (France).
- [10] J-P. Martin, E. Semail, S. Pierfederici, A. Bouscayrol, F. Meibody-Tabar et B. Davat Space vector control of 5-phase PMSM supplied by 5 H-bridge VSIs. Conference on Modeling and Simulation of Electric Machines, Converters and Systems (ElectrIMACS'02), 18-21, Montreal (Canada), (2002).
- [11] H. A. Toliyat, T. A. Lipo, J. C. White. IEEE Transactions on Energy Conversion **6**(4), 679 (1991).
- [12] R. Lyra., T. Lipo, IEEEIAS'01, Chicago, (2001).
- [13] A. Bouscayrol, Ph. Delarue, E. Semail, J. P. Hautier, J. N. Verhille, Revue Internationale de Génie Electrique **5**(3), 431 (2002).
- [14] J. Figueroa, J. Cros, P. Viarouge, Current Control Strategies for Seven Phase Brushless DC Motors ICEM, Brugges, (2002).