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Thermodynamical behaviour of confined many-electron systems

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We discuss application of the fractional dimensionality approach to a variety of low-dimensional systems. Further, we show that thermodynamical behavior of such systems is determined by density of single quasi-particle states, which in turn is characterized by the value of the effective spectral dimension. After presentation, which shows how the spectral dimension is associated with the energy spectrum, we derive formulas for some physical quantities of those systems being an analytical functions of the effective spectral dimension.

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1. Introduction

In the last 15 years the explosion of studies in low-dimensional materials has allowed physicists to discover new electronic phases of matter. The increasing interest to study such systems stems from possible application in the electronics. With the progress in nanotechnology it has been possible to produce a variety of low-dimensional systems such as quantum-wells /QW/, superlattices, quantum wires, quantum dots or fractals. The common feature of all these systems is the constrained motion of the electrons as the result of restricted dimensionality. These constraints induce complicated correlations in the motion of the electrons, and hence (although inter-electron interactions are rather weak) we can classify them as 'Strongly Correlated Electron Systems'.

Provided that heterocomponents of low-dimensional systems show nanoscale modulations, there arises another common feature, at ambient temperatures the mean thermal wavelength becomes comparable to the system size. As a result of the strong localization between interfaces/boundaries there arises confinement of mobile charge carriers, that can alter physical behavior of the initially free electron gas [1]. Therefore it is important to have thorough understanding of the different aspects of confinement in order to be able to design optimized spintronic devices. In this context it is important to find some universal relations between confined geometry of the many body system and the physical phenomena.

The idea of universality stimulates studies of physical behavior in real structures by means of model systems. Within this approach only a few parameters are sufficient to determine the relevant statistical properties of a wide class of systems. Among the most relevant parameters that characterize both single particle and collective behavior of any physical system is the dimensionality. We will show below that thermodynamical behaviour of a low-dimensional system is determined by its spectral (fracton) dimension. If we limit our considerations to the many body system of weakly interacting particles, then its spectral dimension is determined by low-energy excitations. That's why knowledge of both filled and empty eigenstates in close vicinity of the Fermi level becomes crucial for the proper description of the system.

The density of quantum-states in confined many body system is determined by the shape and height of the QW potential barriers. This fact causes that value the effective spectral dimension can differ from the topological (geometrical) dimension that characterizes particle (mass) distribution. In real systems the value of effective spectral dimension extracted from experimental data [2] often shows fractional values. Experiments confirm that fractional dimensionality /FD/ can arise in various laminar system involving different quasiparticles like: polarons, excitons, magnons, phonons and electrons (see [3]-[13] and references therein). The spectral fractality of physical system can be generated two-ways, it can arise due to the fractality of underlying medium (heriditiary fractality due geometrical structure) or due to the fractality of the physical process. The latter mechanism can arise in the vicinity of phase transition when accumulated fluctuations of the order parameter form fractal patterns. In quantum systems under some conditions, e.g. at the critical energy separating localized and extended states, the wave functions are shown to have fractal structure (see [14] and references therein). In the following we will use approximate analytical methods [15], [16] to discuss thermodynamical behaviour of confined many body system. Basing on examples of analytically solvable systems we show that the effective spectral dimension α of low-dimensional systems (i.e., of geometrical dimension $D<3$) can vary in the range $1<\alpha<4$.

2. Fractional spectral dimension

To any physical systems various definitions of dimension can be proposed. In description of collective behavior of many particle system we shall be interested in geometrical dimension, i.e., dimension of the Euclidean space embedding a particle and/or spectral (dynamical) dimension, which is related to the motion of particles within a solid. The spectral dimension α is defined via the density of the free electron/quasiparticle states [15]

$$n(E)dE \approx (E-E_o)^{\alpha/2-1} dE \propto k^{\alpha-1} dk, \quad (1)$$

where the spectral dimension α can take any real (i.e. also fractional) value. In conventional system the spectral dimension is nothing but the dimension of the reciprocal lattice. One can easily check that Eq. (1) reproduces the well-known formulas for the free electron density in 1D, 2D or 3D systems.

In the case of non translation-invariant structures it can been proven that spectral dimension is the best generalization of the Euclidean dimension of the system when dealing with dynamical or thermodynamical properties [17]. To show that, let us consider how the dimensionality enters the thermodynamical quantities. For an ideal Fermi/Bose gas the grand potential reads

$$\ln\Xi = \pm \sum_{k\in\alpha D} \ln(1 \pm e^{-\beta\varepsilon_k}), \quad (2)$$

where the superscript *k* labels different dynamical states of electrons confined within QW. If we replace the summation over quantum states by the integral the formula (2) can be written as

$$\ln\Xi = \int_0^\infty n(\varepsilon)\ln(1 \pm e^{-\beta\varepsilon})d\varepsilon, \quad (3)$$

From Eq. (3) one can easily see that all the information about the dimensionality of the actual system enters thermodynamical formulas *via* the density of states $n(\varepsilon)$. Thus, in view of Eq (1) the thermodynamical evolution of any system depends on its spectral dimension. There is a widespread conjecture that dimensions of the position space (lattice), and of dynamical space (reciprocal lattice) should be both equal and integer. However, there is experimental evidence that in many laminar systems at least one of the abovementioned relations does not hold. In many low-dimensional systems like e.g. superlattices or overlayers, the vibrational as well the electron density of states, extracted from the experimental data correlates with those predicted for the systems of fractional dimension /FD/ [15, 16, 2]. Values of the effective spectral dimension α can be either higher (e.g. $0.86<\alpha=4.7$ in the irradiated GaN structures [18]) or lower (even negative [19]). Moreover in some laminar systems like Ag/Cu(001) overlayer or GaAs/Al_xGa_{1-x}As quantum wells and superlattices as the layer thickness decreases (see [15] and references therein), show (sometime continuous) dimensional crossover from 3D to almost 2D behavior [16]. This effect can be easily understood, in any laminar structure the interlayer tunneling responsible for the charge transfer along the growth direction "z" is the result of thermal fluctuations and has 3D character [4] if

$$k_B T > t_z^2(T)/t_{xy}. \quad (4)$$

In Eq. (4) the t_z and t_{xy} are the interlayer and in-layer hopping rates respectively, while k_B is the Bolzmann constant. When the temperature is lowered to

$$k_B T \approx t_z^2(T)/t_{xy} \quad (5)$$

the interlayer transfer is gradually limited and we have temperature driven dimensional 3D → 2D crossover. Thus, laminar systems offer good testing ground to study the general relations between dimension of a system and physical phenomena.

It is evident from the above, that FD originates from restrained motion of mobile particles or quasi-particles in the stratified media. As it has been shown (see [15] and references therein) , the observed FD of a given physical system is based on physical strength rather than on the geometrical effects. This can be easily understood, numerous physical problems involve basic objects, which are usually described by shrinking or stretching the shape of some characteristic functions. This fact modifies the energy spectrum of the mobile quasi-particles, which in turn determines the spectral dimension [15]. Invoking a FD space in description of such a systems offers a convenient alternative to computational techniques. In this case single parameter - the spectral dimensionality- contains all of the information about the perturbation. We adopt the approach by He [15], who has shown that the anisotropic interactions in 3D space become isotropic ones in lower FD space, where the dimension is the Haussdorff dimension and is determined by the degree of anisotropy. This method allows also application of the k-space formalism. The method by He [15] postulates that the electron quantum states are homogenously distributed in the αD **k**-space and a surface of constant energy is an αD spherical shell. Suppose further that the energy dispersion is parabolic $E-E_o \approx k^2$ we obtain the expression for the density of states in αD **k**-space in the form of Eq. (1) (for details see [15]). This means although the ionic (mass) distribution within a position space of dimensionality β shows no peculiarities, the density of free particle eigenstates shows (sometimes fractional) power law scaling (with effective spectral dimension $\alpha\neq\beta$) [15].

It can be easily proven that definition (1) of the spectral dimension are nothing but the Hausdorff dimension of the effective k-space. Assuming that dynamical space fulfills the connectivity axiom [5], the Hausdorff dimension is equal to that obtained *via* box-counting procedure. Thus, calculation of the Haussdorff dimension of the dynamical space requires determination of the volume $V_E \propto E^{\gamma}$ embedded by a surface of constant energy. It is well-known fact that density of the density of states $n(E)$ is proportional to the measure of the surface of constant energy S_E (i.e., $n(E) \propto S_E$). Moreover volume $dV_E \propto E^{\gamma-1} dE$ of k-space contained between surfaces S_E and S_{E+dE} fulfills relation $dV_E \propto S_E dE$, thus we can write

$$dV_E \propto S_E dE \propto n(E) dE \propto (E-E_o)^{\alpha/2-1} dE \propto k^{\alpha-1} dk \quad (6)$$

This means that $\alpha=\gamma$ and to determine the value of the spectral dimension it suffices to find the exponent that scales the density of states as an function of energy. The latter procedure can be performed provided that energy spectrum is known. This procedure has an advantage over other analytical procedures since scaling exponents in density of states (i.e. spectral dimension) can be also extracted form experimental data [2] . Let us now consider some model systems for which the value of the effective spectral dimension can be derived in an analytical way.

3. Model systems

The effective spectral dimensionality of laminar system can be easily determined provided that energy spectrum of mobile particles within the layer is known. As an example let us consider a semiconductor with planar doping, often used for forming V-shaped potential wells with quasi 2D electron gas.

3.1 V-shaped δ-doped semiconductor

When the deposition of impurities can be represented by the Dirac δ-function, it is called δ doping [13]. The enhanced mobility of the 2D electron gas in V-shaped, δ-doped semiconductor multilayers can be described by the following Hamiltonian [13]

$$H=-\alpha_1 \nabla^4 - \alpha_2 \nabla^2 + V, \quad (7)$$

where

$$\alpha_1 = \frac{\hbar^4}{4E_g}\left(\frac{1}{m^*}-\frac{1}{m_o}\right)^2, \qquad \alpha_2 = \frac{\hbar^2}{2m^*}, \quad (8)$$

while m_o and m^* denote the bare and effective electron mass respectively, while E_g is the bottom of the valence band (for detailed description of the model see [13] and references therein). $V=V(z)$ is the confinement potential that includes electron-electron interaction. At the same time V(z) is the conduction band edge profile. The eigenenergies of the electron/holes are given by [12]

$$\varepsilon_k = \alpha_2 k^2 - \alpha_1 k^4 - \mu, \quad (9)$$

where μ is the Fermi energy, $\vec{k}=(k_x,k_y)$. The ε_k represents the two-dimensional band of the 2D electron gas within the quantum wells of a laminar system. Having the mobile quasi-particle spectrum (9) known we can calculate the density of states as follows [13]

$$n(k)dk \approx \pi(\varepsilon-\varepsilon_o)^{-1/2} d\varepsilon. \quad (10)$$

Expression (10) can be fitted to the general formula (1) by setting $\alpha=1$. This means that we can model such a planar semiconductor with non-parabolic dispersion (3) by a 1D system with parabolic dispersion. In principle it is enough if the density of states fulfills relation (1) in a small energy window close to the Fermi energy. Extensive analytical discussion of how the effective spectral dimensionality is associated with the number of the free electron modes can be found in [3].

3.2 Parabolic quantum well

Modern manipulation opportunities allow fabrication of semiconductor structures with highly controlled variable chemical composition and thickness of layers. Non-square quantum wells involving substantially non-homogenous parts due to deliberate design, rather than having the conventional rectangular profiles, have been studied both experimentally and theoretically in the past few years and found to exhibit attractive performances (see [12] and references therein). The most promising appear the parabolic quantum well /PQW/ systems, which have the ability to absorb light only at the bare harmonic-oscillator frequency irrespective of the electron-electron interaction or the number of electrons in the well. Assuming that the "z" denotes the direction of the planar PQW , the Hamiltonian which describes the electronic structure within the envelope function formalism and effective mass approximation, reads [12]

$$H=-\frac{\hbar^2}{2m}\nabla^2+\frac{D}{2}z^2+E_0 \, . \quad (11)$$

Here, E_o and m_n represent the band gap energy, and the effective mass at the center of the PQW, respectively. D is the curvature of the parabolic potential profile, which is assumed to be infinitely high. In view of the Hamiltonian (11) the electronic states within the PQW are the standard harmonic oscillator states and consequently the electron spectrum is given by [12]

$$\varepsilon_k^n = \frac{\hbar^2 k^2}{2m} + \hbar\omega\left(n+\frac{1}{2}\right) - \mu \, , \quad (12)$$

where μ is the Fermi energy, $\vec{\rho}=(x,y)$ and $\vec{k}=(k_x,k_y)$. The ε_k^n represents the two-dimensional band of the quasi-2D electron gas within the planar PQW. As we can see from Eq. (3) the finite thickness and broken translational symmetry of PQW structure leads to different quantization in the growth direction of the allowed electronic states. Assuming that all conduction electron states below the Fermi level $\varepsilon_F=\varepsilon_{k_F}^{n_F}$ are occupied for both spin states, they fill a (rotational) paraboloide in the (k_x,k_y,n) space. The electronic density of states, taking into account the spin degeneracy, is given by [15]

$$n(E)\,dE=\frac{2m\pi}{\hbar^2}\cdot(E-E_o)\,dE. \qquad (13)$$

Expression (6) can be fitted to the general formula (1) by setting α =4. Conclusion, that the **k**-space of the electron gas confined within the PQW is four-dimensional appears to be counter-intuitive, however as we will show below there are other systems for which the spectral dimension exceeds 4.

3.3 HTC copper oxides

The conjecture that superconductivity comes about because of the fractal structure of underlying medium was raised firstly by Buettner and Blumen [7] in discussion of the high-temperature superconductivity (HTC). In the HTC copper oxides the onset of superconductivity is closely related to the oxygen deficiency. It was postulated that the oxygen vacancies located mainly within the CuO2 planes of the YBCO system form a fractal structures. Since the fraction vibrational frequency cutoff wFD is much greater than the Debye frequency of crystalline systems there arose conjecture that conduction electron scattering off fractons can be responsible for the high critical temperature [11]. Suppose that HTC SC is in the normal phase, then the energy spectrum of the mobile, bipolaron local electron pairs /LEP's/ is given by $\varepsilon_k=\gamma|k|$ [20] where $\vec{k}\in(k_x,k_y)$ space. Thus the density of the LEP's pairs is given by $n(k)dk\propto kdk$ or equivalently $n(E)dE\propto|E-E_o|dE$. The latter formula means that the effective spectral dimension of the LEP's system equals 4. Although all the discussed model systems are the quasi-2D (when geometrical meaning of dimensionality is used) their spectral dimension varies within $1<\alpha D<4$ range. Since collective behavior of the many body system is determined by density of states this means that these quasi-2D systems should behave in different manner. Below we will discuss some consequences of the variable spectral dimensionality of layered systems.

4. Condensation of the local electron pairs in the HTC superconductors

The conventional theory of boson condensation derived for systems of integral dimensionality [8] can be easily extended onto systems, which exhibit fractional spectral dimension α. The total number of bosons $N_B(T)$ in the system consists of the $N_{B,0}(T)$ ones that occupy the ground state ε_o,(ε_o=0 in the thermodynamic limit), while the others are distributed over higher energy levels. In view of this we have: [8]

$$N_B=N_{B,0}(T)+\sum_{\mathbf{k}\neq 0}\frac{1}{e^{\beta(\varepsilon_k-\mu_B)}-1}, \qquad (14)$$

where $\beta=1/K_BT$ and $\mu_B<0$ is the chemical potential. In Eq. (14) we assume that summation goes over the **k** -states filling the fractional αD space. After some algebra we can find the condensate fraction $N_{B,0}(T)/N_{B,0}(0)$ as [4].

$$\frac{N_{B,0}(T)}{N_{B,0}(0)}=1-\frac{V_\alpha}{N_B}\left(\frac{m^*}{2\pi\beta\hbar^2}\right)^{\alpha/2}\zeta(\alpha/2). \qquad (15)$$

The condensate fraction falls off when the temperature is increased and eventually at T_c the condensate vanishes i.e. $N_{B,0}(T)/N_{B,0}(0)=0$. From this condition we can derive the formula for the critical temperature T_c as a function of the effective spectral dimension α.

$$T_{c,\alpha}=\frac{m^*}{2\pi k_B\hbar^2}\left(\frac{V_\alpha}{N_B}\zeta(\alpha/2)\right)^{2/\alpha}. \qquad (16)$$

The fact that the phase transitions are governed by the value of spectral rather than spatial dimension has been established long time ago when studying fractal systems [9], [10]. However, in the case of bulk or laminar systems phase transitions still are improperly classified according to the value of their spatial (geometrical) dimension.

Let us discuss some consequences of Eq. (16). In conventional theories the ratio V_α/N_B is treated as the inverse boson pair concentration n_B^{-1}. Such interpretation is justified provided that spectral dimension α and dimension of real space β (position space) are equal. However, in systems of FD such interpretation is not valid. Suppose, that in the system under consideration we have some characteristic length L, then the volume $V_\alpha\approx L^\alpha\approx(k_F)^{-\alpha}$. Simultaneously the volume of the system, i.e. volume filled with quasi-particles (boson pairs) can be expressed as $V_\beta\approx L^\beta$. In view of this, concentration n_B being the real space quantity reads as $n_B=N_B/V_\beta$. Distinction between this different notions of dimensionality is often missed, but as it will be shown below crucial in proper description of dimensional effects in SC. Inserting relation $V_\alpha \approx L^\alpha$ into Eq. (15) we have:

$$T_{c,\alpha} = \frac{m^* L^2}{2\pi k_B \hbar^2} \left(\frac{\zeta(\alpha/2)}{N_B} \right)^{2/\alpha}. \tag{17}$$

In many layered systems when external conditions (temperature, thickness or fields) change the FD system undergoes (sometimes continuous) dimensional crossover. Let us consider a FD system in two states, which exhibit spectral FD α and α' respectively. Moreover let us assume that number of preexisting boson pairs is constant during this dimensional crossover. In view of Eq. (17) the hypothetical critical temperatures in both states fulfill the relation.

$$\frac{T_{c,\alpha'}}{T_{c,\alpha}} = \frac{m^*_{\alpha'}}{m^*_\alpha} \frac{\zeta(\alpha'/2)^{\alpha'/2}}{\zeta(\alpha/2)^{\alpha/2}} N_B^{(2/\alpha'-2/\alpha)}. \tag{18}$$

Let us study the variation of the critical temperature $T_{c,\alpha'}$, associated with the continuous dimensional crossover. We assume, that in Eq. (18) α=3, i.e. we take the 3D case as the reference system. From Eq. (18) results that the critical temperature is elevated when the spectral dimensionality increases. The ratio (18) can be by an order of 10 or even larger when the spectral dimension α' exceeds 3. This means that in this situation the condensation occurs just at the pair formation temperature.

5. Layered magnetic systems

Another collective magnetic phenomenon governed by the low-energy excitations is the RKKY exchange interactions. Let us assume that some localized magnetic moments are immersed in a sea of electrons which exhibit fractional spectral dimensionality α. As the result of electron, spin dependent scattering there arises an oscillatory polarization of the free electron system being a source of the RKKY interaction.

5.1 RKKY interaction in a system of fractional dimension

Within perturbative approach, the RKKY interaction between magnetic moments of the magnetic ions (μ_i and μ_j)located at a distance R_{ij} can be written as [3]

$$H(R_{ij}) = \frac{1}{2} A^2 \chi(R_{ij}) \, \vec{\mu}_i \, \vec{\mu}_j, \tag{19}$$

where $\chi(r_{ij})$ is the non-uniform static susceptibility and A is a constant. The explicit form of the $\chi(R_{ij})$ is given by [3]

$$\chi(R) = -T \sum_l G(i\omega_l, R)^2, \tag{20}$$

where $\omega_l = \pi T(2l+1)$ are the Matsubara frequencies and the electronic Green's function is

$$G(i\omega, \vec{R}) = \int d^\alpha \vec{k} (2\pi)^\alpha \cdot \frac{e^{i\vec{k}\cdot\vec{R}}}{i\omega - \varepsilon_k}. \tag{21}$$

As we can see the calculation of the RKKY exchange integrals reduces to integration over α*D* free electron eigenstates with density of states given by Eq. (7).

Contour integration over ω in eq. (20), accounting for the discontinuity of $G(i\omega, R)$ at ω=0, in the low temperature limit, leads to an expression for the RKKY exchange integral in an α*D* system (compare [3])

$$\chi(r) \approx \frac{\chi_o}{r^\alpha} r^2 \left[J_{\alpha/2-1}(x) \, Y_{\alpha/2-1}(x) + J_{\alpha/2}(x) \, Y_{\alpha/2}(x) \right]. \tag{22}$$

with $Y_\nu(x)$ being the Neumann function, $x = k_F r$ and α=1 is the effective spectral dimension of the system under consideration.

The calculated static susceptibility that determines exchange integral $J(r) = J_o \cdot \chi(r)$ of the RKKY interaction (22) in the PQW system, shows conventional, sign-reversal oscillatory behavior with the period governed by the wave-vector $2k_F$. It is well known fact that the leading term of the RKKY exchange integrals decays with the interspin separation as r^{-D} (see [3] and references therein), where *D* is the dimension of the system. The result (22) indicates clearly, that the spatial variation of the envelope function of $\chi(r)$ is governed by the spectral dimension α. The relation α<β is the direct consequence of dispersion (3), which generates lower density of states $n(\varepsilon)$ compared to that of conventional parabolic dispersion. The fact that in the case of our system α<β means that the RKKY exchange integrals fall off (with the interspin separation) in a different manner that one would expect for the quasi-2D systems. We should note here that one would expect that the formulae for the RKKY range function (22) refers also to the Friedel oscillations in low-dimensional systems.

5.2 Thermodynamical behavior

The contribution from magnonic excitations in laminar system (that can be described by fractional dimension) to the internal energy of the system that is given by [21]

$$U=\int_0^\infty \frac{\tilde{\varepsilon}}{e^{\tilde{\varepsilon}/kT}-1}dn(\tilde{\varepsilon})=\int_0^\infty \frac{\tilde{\varepsilon}^{\alpha/2}}{e^{\tilde{\varepsilon}/kT}-1}d(\tilde{\varepsilon})=b\,T^{\alpha/2+1}. \quad (23)$$

where $\tilde{\varepsilon}=\varepsilon-\varepsilon_0$. Result (23) predicts arbitrary fractional power law temperature dependence of the specific heat $C_m \propto T^{\alpha/2}$, while in case of bulk systems it should be $C_m \propto T^{3/2}$. It is worth to note that numerical simulations by Ilković and Tuleja [22] predict similar behavior of C_m in layered system with dimensionality α that varies, as the function of the QW thickness, within $2.2<\alpha<3.0$ range. Similarly, the Bloch exponent in the low temperature magnetization $M(T)$ can be calculated giving us $M(T)=M(0)(1-BT^{\alpha/2})$.

6. Fractal systems

Last years the fractal systems attracted much attention. The fractality of physical systems manifests itself many ways, it can arise from the geometrical structure of nanocomposites [23] or diffusion limited aggregates (hereditary fractality), domain patterns [24] or fractality of relaxation processes [25]. The essential property of fractal systems (and consequently of physical processes on them) is a hierarchical organization of its elements, described by discrete scaling laws, which makes the fractal, regardless of magnification or contraction scale, looks the same. This property of fractals is called self-similarity, self-affinity or self-replicability. The fractal structure characterizes the static properties of the system while the thermodynamical description the information about spectral dimension is necessary. Therefore we should recall dynamical properties and interactions on fractal systems. The fractality influences the dynamical behavior two-ways. The first dependence comes from fractal distribution of interacting species in the embedding Euclidean space (mass fractal), while the second stems from the boundary conditions set on fractal perimeters (fractal drums). Consequently, evolution of fractal systems a rule involves fractional dynamics of the system [26], which in turn involves fractional calculus as the main tool of the theoretical description. The elementary excitations are the solutions of fractional differential equation e.g. of the wave-like form [26], [27], [28].

$${}_{\xi}D^{\beta}u-\frac{1}{c^{\alpha}}\,{}_{t}D^{\alpha}u=0 \quad (24)$$

In Eq. (24) ${}_{\xi}D^{\beta}u$ and ${}_{t}D^{\alpha}$ denote the operators of fractional order differentiation with respect to the space and time variables (the Remann-Louville, Riesz, Caputo etc. derivatives, depending on the type of physical process, for details see [29])) with respect to space or time variables. The fractional time derivative accounts for the random interactions with the surrounding, while fractional space derivatives describe the reduced dimensionality of the system. We should remind the reader here that parameters α and β model different features of the dynamical system. The order of fractional time derivative α describes the damping, while the value of β reflects the reduced dimensionality of the fractal set. The direct consequence of the unconventional dynamics of the system is that the value of the spectral dimension as the rule is a fraction.

7. Summary

We study the many body system, in which the mobile charge carriers are confined within quantum-wells. Since thermodynamical behavior of such systems is determined by the low-energy excitations of single quasiparticles to account the effects of quantum confinement it suffices to know the density, of states close to the Fermi level. In our study we limit ourselves to the case, when the density of single quasiparticle states is given by Eq. (1). The effective density of states shows simple scaling relation with an exponent (spectral dimension) α. We show that the effective spectral dimension of quasi-2D system can be a fraction from the $1<\alpha<4$ region, and can vary when external parameters like temperature or pressure are changed. When applied to real systems like magnetic superlattices or HTC copper oxides the effective dimensionality approach allows rigorous calculation of analytical expressions for some physical quantities. Among others we have derived formulae for the interlayer coupling in magnetic superlattice and critical temperature in LEP based superconductor as an analytical functions of spectral dimension.

In conclusion we can state that presented approach offers convenient alternative to the numerical simulations and gives better insight into the physics of the low-dimesional systems. From the discussion above it is evident that topological (geometrical) dimension characterizes static properties of a physical system while the spectral dimension describes its dynamical evolution. The fact that effective spectral dimension depends on the external fields indicates the way in which physical properties of quasi-2D systems can be manipulated form outside. It is evident that potential applications of the results obtained cover much wider variety of many body systems than those discussed above.

References

[1] P. Bruno: Phys. Rev. B **43** (1995) 411.

[2] D. Mo, Z.J. Jiang and N. Ke: Solid State Commun. **114**, (2000) 277.

[3] Z. Bak, R.Jaroszewicz, W.Gruhn: J. Mag. Mag. Mater. **213** (2000) 340.

[4] Z. Bak: Phys. Rev. **B68** (2003) 064511.

[5] F.H. Stillinger: J. Math. Phys. **18** (1977) 1224.

[6] M. Casas, M. de Llano, A. Puente, A. Rigo and M. A. Solís : Solid State Commun. **123** (2002) 101.

[7] H. Buettner, A. Blumen, Nature **329**, 700 (1987).

[8] M. Casas, A. Rigo, M. de Llano, O. Rojo and M. A. Solis: Phys. Lett. A **245,** 55 (1998).

[9] P. Buonsante, R. Burioni, D. Cassi, A. Vezzani: Phys. Rev. B **66,** 094207 (2002).

[10] Yu Shi, Changde Gong: Phys. Rev. B. **49,** 99 (1994).

[11] Jiang Q, De-Cheng Tian, Jian-Xin Li, Zheng-You Liu, Xiao-Bing Wang, Zhe-Hua Zhang Phys. Rev. **B48**, 524 (1993).

[12] Z. Bak: Solid State Commun. **118,** 43 (2001).

[13] Z. Bak: Czech. J. Phys. **52,** A25 (2002).

[14] A. Mildenberger, F. Evers, A. D. Mirlin, Phys.Rev. **B66**, 033109 (2002).

[15] X.F. He: Solid State Commun. **75,** 111 (1990).

[16] X. F. He: Phys. Rev. B **43,** 2063 (1991).

[17] Z.Bak, Phase Transitions **80,** 79 (2007).

[18] E. Gaubas, P. Pobedinskas, J. Vaitkus, A. Uleckas, A. Žukauskas, A. Blue, M. Rahman, K.M. Smith, E. Aujol, B. Beaumont, J.-P. Faurie and P. Gibart, Nucl. Instrument. @ Methods **A552**, 82 (2005).

[19] B. B. Mandelbrot , J Stat. Phys.**110** 739 (2003).

[20] R. Micnas, J. Ranninger, S. Robaszkiewicz: Rev. Mod. Phys. **62,** 113 (1990).

[21] W. Gruhn, Czech.J. Phys., **54,** D225 (2004).

[22] V. Ilković, S. Tuleja: Czech. J. Phys. **52,** A57 (2002).

[23] S. N. Gvasaliya, S. G. Lushnikov, Y. Moriya, H. Kawaji, T. Atake, Physica **B305**, 90 (2001).

[24] B. Tadic, Eur. J. Phys. **B28**, 81 (2002).

[25] N. M. Galyarova, S. V. Gorin, L. I. Dontsova, Mat. Res. Innovat. **3**, 30 (1999).

[26] Z. Bak, Materials Science-Poland, **27**, 491 (2007).

[27] Ya. E. Ryabov, A. Puzenko, Phys. Rev. **66B**, 184201 (2002).

[28] Y. Z. Povstenko, Int. J. Eng. Sci. **43**, 977 (2005).

[29] A. A. Kilbas, H. M. Srivastava, J. J. Trujillo: "Theory and Applications of fractional Differential Equations", Elsevier, Amsterdam 2006.

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