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Tunnelling properties of two-well nanostructures with δ-like barriers placed into the strong electromagnetic fields

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The theory of photon-assisted transport of electrons through the two-well resonant tunneling structure with δ-like potential barriers driven by electromagnetic field is developed. The transmission coefficient for the nano-structure is obtained using the exact solution of time-dependent one-dimensional Schrodinger equation, taking into account the electron-photon interaction.

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1. Introduction

The rapid progress in THz devices [1, 2] operating at the basis of nano-size resonant tunneling structures (RTS) facilitates the constant experimental [3-6] and theoretical [7-17] research of photon-assisted transport (PAT) of electrons though the nano-structures driven by electromagnetic fields.

The theory of nonlinear electronic transport through the structures with time-periodical potentials is developed within different approaches [18, 19]. In the wide group of papers the model tunneling Hamiltonians, written in the representation of second quantization [10-13] were used together with the well developed mathematical techniques, such as density and scattering matrix, nonequilibrium Green's-Keldysh functions and other. The main advantage of tunneling Hamiltonian is that it can take into account different dissipative subsystems: phonons, impurities, electron-electron interaction and so on. It gives opportunity to study the ballistic transport of electrons and distinguish the role of dissipative processes in nonlinear transport phenomena.

The second group of theoretical papers [7-9, 14-17] is based at the Hamiltonians obtained in the coordinate representation. They do not contain any fitting parameters and are characterized by physical and geometrical parameters of nano-structures.

Studying the PAT of electrons, the Floquet method [18, 19] is usually used together with transfer-matrix, S-matrix, classic Green's functions, times-dependent perturbation theory and so on. In order to simplify the analytical and numerical calculations considering that the interaction between electrons and electromagnetic field is actual inside of the nanostructure only, the one-dimensional model is used as a rule. The interaction in it is usually transformed to the time-dependent potential: $U(z)\cos\omega t$.

In this paper, we develop the theory of nonlinear PAT of electrons through the open two-well RTS driven by the periodical electromagnetic field, exactly taking into account the electron-field interaction in the Hamiltonian of the system. We obtain the exact solutions of the complete Schrodinger equation for all parts of the structure.

2. Transmission coefficient for the two-well RTS driven by electromagnetic field

We study the transport properties of two-well RTS placed into the outer medium and driven by the homogeneous electromagnetic field $\mathcal{E}(t) = 2\mathcal{E}\cos\omega t$ characterized by the intensity $\mathcal{E}$ and frequency ω. In the Cartesian coordinates with the beginning at the left interface of the input barrier, fig. 1, and $0z$ axis perpendicular to the planes of nano-films, the electron moves along the $0z$ axis from the left to the right through the nano-structure.

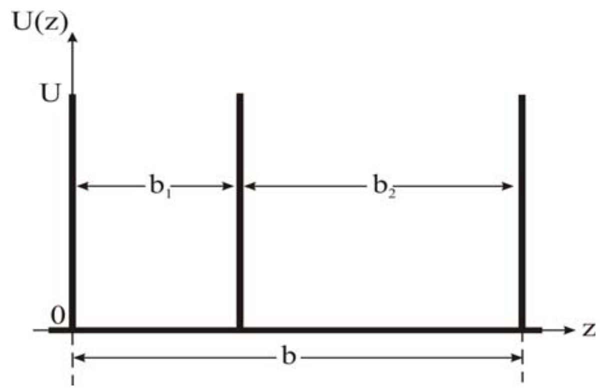

Fig. 1. Potential energy scheme for the electron and geometry of two-well RTS.

Neglecting the scattering inside of the RTS, the time-dependent one-dimensional Schrodinger equation for the electron is written as

$$i\hbar\frac{\partial\Psi(z,t)}{\partial t}=[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial z^2}+U(z)+H_{\text{int}}+ +\frac{2(e\epsilon)^2}{m\omega^2}\sin(\omega t)^2]\Psi(z,t). \quad (1)$$

The complete Hamiltonian of the electron-photon system in eq. (1) contains the kinetic energy of electron (the first term), its potential energy written in typical δ-barrier approximation [16, 17]:

$$U(z)=U\Delta_1\delta(z)+U\Delta_2\delta(z-b_1)+U\Delta_3\delta(z-b)\,, \quad (2)$$

linear over the intensity term describing the electron-electromagnetic field interaction

$$H_{\text{int}}=-2e\epsilon\cos(\omega t)\{z[\theta(z)-\theta(z-b)]+b\theta(z-b)\} \quad (3)$$

and square over the field the fourth term, where: e, m – charge and mass of pure electron; U, $\Delta_{(1,2,3)}$ – height and widths of potential barriers, respectively; b_1 – the width of input potential well; b – the size of two-well RTS.

The eq. (1) in the inner parts of nano-structure ($0<z<b$) has two exact linearly independent solutions

$$\psi^{\pm}(E,\omega,z,t)=\exp\{\pm ik_0(z+\frac{2e\epsilon}{m\omega^2}\cos(\omega t)) -\frac{i}{\hbar}[Et-\frac{2e\epsilon z}{\omega t}\sin(\omega t)+ +2\frac{(e\epsilon)^2}{m\omega^2}(1-\frac{\sin(2\omega t)}{2\omega t})t]\} \quad (4)$$

which describe the forward and backward waves with quasi-momentum $k_0=\hbar^{-1}\sqrt{2mE}$. Outside of the RTS the solutions are evidently simplified [17].

The exact wave function of the whole system, as linear combination of both solutions (4), taking into account the expansion of all time-periodical functions into the exact Fourier ranges and considering the superpositions with all field harmonics is written as

$$\Psi(E,\omega,z,t)=\sum_{p=-\infty}^{+\infty}[\psi^+(E+p\Omega,\omega,z,t)+ +\psi^-(E+p\Omega,\omega,z,t)]\,, \quad (5)$$

where

$$\psi^{\pm}(E+p\Omega,\omega,z,t)=e^{-\frac{i}{\hbar}(E+p\Omega)t}\frac{\sin(2\pi\alpha\beta^2)}{\pi}\times \quad (6)$$

$$\sum_{n_1,n_2=-\infty}^{+\infty}\frac{(-1)^{n_1}j_{n_2}(\alpha\beta^2)}{n_1+2\alpha\beta^2}f(\pm k_p,\omega,z,t)e^{\pm ik_pz+i(n_1+2n_2)\omega t}\,,$$

$$f(\pm k_p,\omega,z,t)=A^{\pm}_{0,p}\theta(-z)+ +\sum_{s=1}^{2}\sum_{\substack{n_3,n_4\\ n_5,n_6=-\infty}}^{+\infty}A^{\pm}_{s,p}\frac{\sin(2\pi\alpha\beta^2)}{\pi}\frac{i^{2n_3\pm n_5}e^{i(n_3+2n_4\mp n_5+n_6)\omega t}}{n_3+2\alpha\beta^2} \times j_{n_4}(\alpha\beta^2)j_{n_5}(4\alpha\beta bk_p)j_{n_6}(2\beta\frac{z}{b})[\theta(z-z_{s-1})- \theta(z-z_s)]+A^{\pm}_{3,p}\sum_{n_3=-\infty}^{+\infty}j_{n_3}(2\beta)e^{in_3\omega t}\theta(z-b). \quad (7)$$

The convenient denotations:

$$k_p=\hbar^{-1}\sqrt{2m(E+p\Omega)}\,;\quad k_b=b^{-1}\,;\quad U_b=e\epsilon b\,;$$
$$\alpha=\frac{\hbar^2k_b^2}{2m\Omega}\,;\qquad \beta=\frac{U_b}{\Omega} \quad (8)$$

have the evident physical sense: α – dimensionless kinetic energy of electron with quasi-momentum k_b and β – dimensionless potential energy of electron-field interaction written in the units of electromagnetic field energy $\Omega=\hbar\omega$.

All unknown coefficients ($A^{\pm}_{s,p}$) are definitely fixed by the conditions of continuity of wave functions and their densities of currents at all interfaces in any moment of time:

$$\Psi(E,\omega,0-\eta,t)=\Psi(E,\omega,0+\eta,t)\,,\quad(\eta\to0)$$
$$\left.\frac{\partial\Psi(E,\omega,z,t)}{\partial z}\right|_{z=0+\eta}-\left.\frac{\partial\Psi(E,\omega,z,t)}{\partial z}\right|_{z=0-\eta}= =k_b\left(\frac{U\Delta_1k_b}{\alpha\Omega}+4i\beta\sin(\omega t)\right)\Psi(E,\omega,0,t)\,, \quad (9)$$
$$\Psi(E,\omega,z_s-\eta,t)=\Psi(E,\omega,z_s+\eta,t),\quad(s=1,2)$$
$$\left.\frac{\partial\Psi(E,\omega,z,t)}{\partial z}\right|_{z=z_s+\eta}-\left.\frac{\partial\Psi(E,\omega,z,t)}{\partial z}\right|_{z=z_s-\eta}= =\frac{k_b^2U\Delta_{s+1}}{\alpha\Omega}\Psi(E,\omega,z_s,t).$$

These conditions must be fulfilled for each separate harmonic (p). We assume that the electron transports through the main canal (p=0), thus: $A^{+}_{0,p=0}\neq0$, $A^{+}_{0,p\neq0}=0$ and $A^{-}_{3,p}=0$ because the reflected wave is absent outside of the RTS.

The system of eq. (9) contains the infinite number of equations respectively $A_{s,p}^{\pm}$ coefficients due to the infinite number of harmonics. However, it can be confined by the demanded rather big number of positive (N^{+}) and negative (N^{-}) harmonics during the numerical calculations.

The inhomogeneous system of eqs. (9) allows to obtain all $A_{s,p}^{\pm}$ coefficients throughout $A_{0,0}^{+}$ at the finite number of harmonics. The latter is found from the normality condition for the complete wave function of the whole system

$$\int_{-\infty}^{+\infty} \Psi^{*}(k_0,\omega,z,t)\Psi(k_0',\omega,z,t)\,\mathrm{d}z = \delta(k_0 - k_0') \,. \quad (10)$$

Considering the electromagnetic field, the density of current in any moment of time is written within the known expression [14]

$$j(E,z,\omega,t) = \frac{\mathrm{i}\hbar}{2m}\left[\Psi(E,\omega,z,t)\frac{\partial}{\partial z}\Psi^{*}(E,\omega,z,t) - c.c.\right]$$

$$-\frac{2e\mathcal{E}}{m\omega}\sin(\omega t)\,|\,\Psi(E,\omega,z,t)\,|^{2} \,. \quad (11)$$

After calculation of the densities of currents in forward (j^{+}) and backward (j^{-}) directions at the entrance ($z = 0$) and exit ($z = b$) of RTS, the time-independent transmission coefficient $D(E,\omega)$ is obtained from the ratio between the densities of currents j_0^{+} and j_3^{+}, averaged over the period of time T

$$D(E,\omega) = \langle j_3^{+}(E,\omega)\rangle\,\langle j_0^{+}(E,\omega)\rangle^{-1} \quad (12)$$

where

$$\langle j_s^{+}(E,\omega)\rangle = \frac{1}{T}\int_0^T j_s^{+}(E,\omega,t)\,\mathrm{d}t \quad (13)$$

The obtained transmission coefficient (12) for the two-well RTS allows us to calculate the resonance energies and widths of main and arbitrary number of satellite quasi-stationary states of electron, interacting with electromagnetic field of arbitrary intensity ($\mathcal{E}$) and frequency (ω). The developed theory proves that this interaction, besides the renormalization of pure quasi-stationary states of electron, brings to the appearance of new mixed states, which produce the specific transmission canals of two-well RTS.

4. The properties of transmission canals of two-well RTS driven by electromagnetic field

It is well known [20], that without the electromagnetic field the resonance energies (E_n) of electron quasi-stationary states in RTS with the Hamiltonian like: $H = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial z^2} + U(z)$ can be calculated using different techniques: S-matrix, probability distribution function, transmission coefficient. The same techniques should be used for the calculation and analysis of electron-photon quasi-stationary spectrum in RTS driven by electromagnetic field of arbitrary intensity ($\mathcal{E}$) and frequency (ω).

We observe the typical, often experimentally [1, 2, 5] and theoretically [19-21] studied two-well RTS ($In_{0.52}Al_{0.48}As/In_{0.53}Ga_{0.47}As$) with known geometrical parameters: $b = b_1 + b_2 = 9\,nm$, $\Delta_1 = 12\,nm$, $\Delta_2 = 9\,nm$, $\Delta_3 = 12\,nm$ and physical ones: $U = 516\,meV$, $m = 0.043\,m_0$, m_0 – is the mass of pure electron. We calculate the resonance energies of electron-photon system using the theory for the transmission coefficient $D(E,\omega)$ developed in the previous Section. The magnitudes of the resonance energies are fixed by the respective positions of D maxima in the scale of electron energies E.

In fig. 2 the resonance energies of electron-photon quasi-stationary states are shown as functions of the position of inner potential barrier between two outer ones (b_1) of RTS driven by electromagnetic field with non-resonance energy $\Omega = 153\,\mathrm{meV}$ and such intensity ($\mathcal{E}$) which provides the driving potential energy $U_b = 20\,\mathrm{meV}$ in the RTS with the size $b = 9\,nm$.

Fig.2 shows that the spectrum contains the renormalized main electron quasi-stationary states with the energies $E_{n(p=0)}$ and the states which are the superpositions of electron ones and the respective number of satellite field harmonics with the energies $E_{n(p)} = E_n + p\Omega$, $p \neq 0$ which appear due to the interaction with the electromagnetic field. It is clear that the behaviour of resonance energies of main and satellite states is qualitatively similar.

The resonance energy of $|n(p)\rangle$ state non-monotonously depends on b_1, displaying the ascending and descending plots and containing n maxima and $n+1$ minimums. The analysis of the curves allows to distinguish the region of maximal location of electron in $|n(p)\rangle$ state inside of RTS at arbitrary b_1. If the resonance energy is at the ascending plot, then the electron with maximal probability is located inside of the output potential well with the width b_2. If it is at the descending plot, then the electron with maximal probability is located inside of the input potential well with the width b_1.

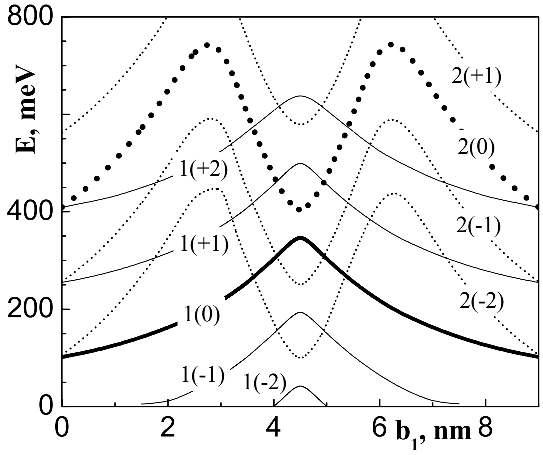

Fig. 2. Resonance energies of main $|n(0)\rangle$ and satellite $|n(p\neq 0)\rangle$ quasi-stationary states of electron-photon system in two-well RTS as function of b_1 at $U_b=20\,meV$; $\Omega=153\,meV$; $b=9\,nm$; $\Delta_1=12\,nm$; $\Delta_2=9\,nm$; $\Delta_3=12\,nm$.

Fig. 2 also proves that the anti-crossings between main $|n(0)\rangle$ and satellite $|n+1(0)\rangle$ and between satellite $|n(p)\rangle$ and $|n+1(p)\rangle$ quasi-stationary states are observed in the symmetrical two-well RTS ($b_1=b_2=4.5$ nm). The magnitudes of anti-crossings between the resonance energies of electron-photon states are determined by the relationships between the widths of inner (Δ_2) and outer (Δ_1, Δ_3) potential barriers, in analogy with paper [20] where the electromagnetic field is not considered. The collapse of anti-crossing states happens at $\Delta_2>\Delta_1+\Delta_3$, when the distance between both wells becomes rather big.

The PAT of electrons through the one-well RTS was studied in refs. [18-21] in resonance fields with the energies: $\Omega=\Omega_{n,n-1}=E_n-E_{n-1}$. The new phenomena concerning the transmission properties of nano-structure were observed: the increasing field intensity caused the splitting of the maximum of transmission coefficient into two ones. Therefore, we are going to study the behaviour of transmission coefficient for the two-well RTS at the resonance energies of electromagnetic field.

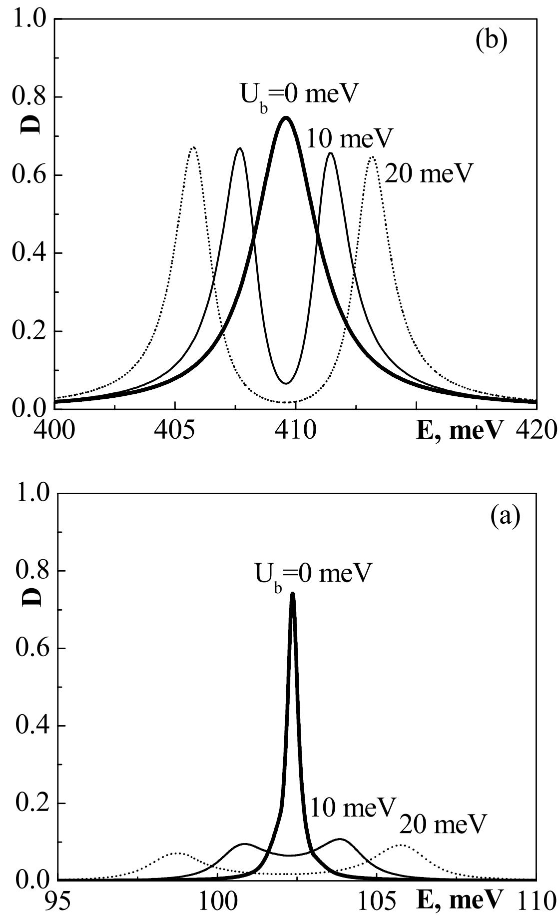

Fig. 3. Transmission coefficient (D) as function of electron energy (E) in the vicinity of the energies E_1 (a) and E_2 (b) at the resonance energy of electromagnetic field: $\Omega_{21}=307.2$ meV and driving potentials: U_b=0, 10, 20 meV for RTS with geometrical parameters: b_1=0 nm; b_2=9 nm.

The transmission coefficient (D) as function of electron energy (E) for the three geometrical configurations of two-well RTS: $b_1=0$ nm (fig.3), $b_1=b/4=2.25$ nm (fig.4), $b_1=b/2=4.5$ nm (fig.5) and three magnitudes of driving potential energy: $U_b=0, 10, 20$ meV is calculated. The resonance energy of electromagnetic field is: $\Omega=\Omega_{21}=E_2-E_1$ (figs.3-5). It corresponds to the difference between the energies of "pure" quasi-stationary states of electron.

Figs. 3-5 prove that the dependence of transmission coefficient, shown in the vicinity of resonance energies of first (fig.3a, fig.4a, fig.5) and second (fig.3b, fig.4b, fig.5) main electron-photon quasi-stationary states is qualitatively similar for the three geometrical configurations of RTS.

When the field is absent ($U_b = 0\,\text{meV}$) the transmission coefficient $D(E)$ has the shape of Lorentz curve. Herein, the best transmission properties ($\max D \approx 1$) are observed for the symmetrical two-well RTS (fig.5).

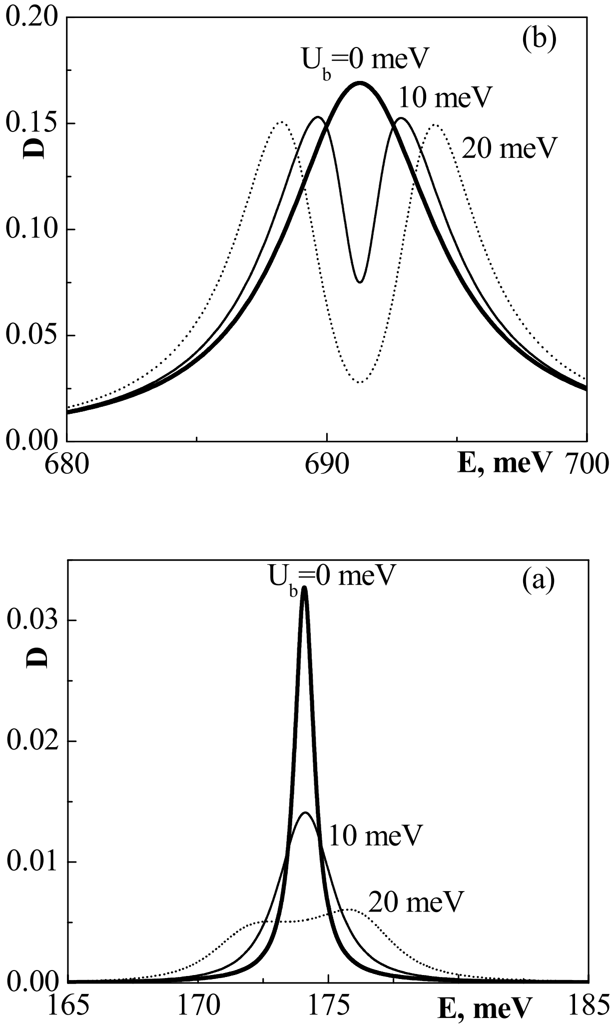

Fig. 4. Transmission coefficient (D) as function of electron energy (E) in the vicinity of the energies E_1 (a) and E_2 (b) at the resonance energy of electromagnetic field: Ω_{21}=517.2 meV and driving potentials: U_b =0, 10, 20 meV for RTS with geometrical parameters: b_1 =2.25 nm; b_2 =6.75 nm.

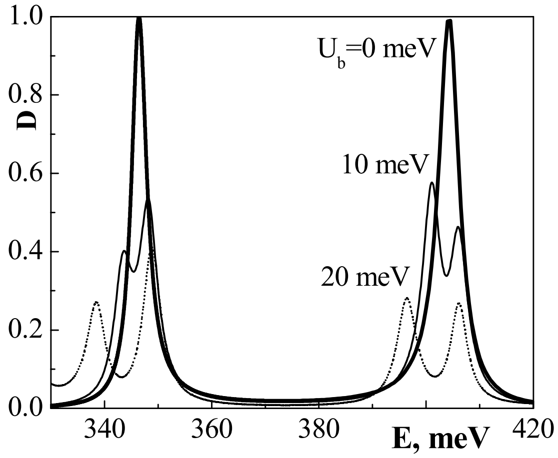

Fig. 5. Transmission coefficient (D) as function of electron energy (E) in the vicinity of the energies E_1 and E_2 at the resonance energy of electromagnetic field: Ω_{21}=57.8 meV and driving potentials: U_b =0, 10, 20 meV for RTS with geometrical parameters: b_1 =4.5 nm; b_2 =4.5 nm.

When the driving potential (U_b) appears and becomes stronger, the Lorentz curve is deformed, its maximum decreases and $D(E)$ takes the shape of two-humped curve with two Lorentz-like peaks with increasing distance between them. The appearance of two maxima in the vicinity of $E_{1(0)}$ energy (figs.3a, 4a, 5) is caused by the superposition of first main quasi-stationary state $|1(0)\rangle$ and first negative satellite of the second main quasi-stationary state $|2(-1)\rangle$. In the vicinity of $E_{2(0)}$ energy (figs.3b, 4b, 5) the same is caused by the superposition of second main quasi-stationary state $|2(0)\rangle$ with the first positive satellite of the first main quasi-stationary state $|1(+1)\rangle$. This pair of maxima produces two canals of almost equal transmission for the RTS. The transmission of two canals in the vicinity of $E_{2(0)}$ energy for the asymmetrical one-well RTS (fig.3) and two-well RTS with narrower input potential well (fig.4), is much bigger than the transmission of two canals in the vicinity of $E_{1(0)}$ energy.

Analysis of figs. 3-5 brings to the conclusion: independently of the position of inner barrier, in the vicinity of electron resonance energies E_1 and E_2 at the field resonance energy $\Omega_{21} = E_2 - E_1$, the transmission of both main canals decreases and each of them splits into the pair of new canals with decreasing transmission when the driving potential U_b becomes stronger. The both pairs of canals display the maximal transmission for the symmetrical two-well RTS.

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