Vortex structures in optical fibers with spatial dependence of the refractive index

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It is well-known that vortex structures of laser beam can be created by different optical masks and holograms. These vortices are solutions of the 2D scalar Leontovich equations and admit amplitude and phase singularities.
The main idea of present work is to investigate the formation of vortex structures for optical pulses, evolving in dispersive Kerr-type nonlinear medium with spatial dependence of the refractive index. The propagation of such type of laser pulses is governed by nonlinear vector system of amplitude equations. We found new class of analytical solutions with vortex structures for concave gradient fibers. Their stability is a result of the balance between diffraction and nonlinearity, as well as the balance between nonlinearity and angular distribution.

1. Introduction

In the last few decades, there was a considerable progress in the investigation of the nonlinear evolution of short laser pulses in different optical fibers. This is a result of the rapid development of the scientific knowledge and new technologies in the field of nonlinear optics with a huge number of applications in high intensity laser systems in nuclear physics and medicine for better diagnostic and therapeutic methods with high resolution.

It is important to mention that laser beams in linear regime of propagation admit different types of phase changes that lead to singularity. Such types of light beams are called *singular.* This uncertainty is a result of the phase jump and determines a zero-intensity region at a specific location in the beam. Thus, vortex structures are created. Optical vortices are phases of singular structures providing a wide range of applications in optical cryptography, quantum computers for encoding and recording of information, making optical tweezers, etc. As we already pointed out, the condition for obtaining such kind of structures is the right balance between diffraction and nonlinearity. It is well known that different types of vortex structures of laser radiation can be created by optical holograms and various optical masks [1-6]. Optical vortices appear in the plane perpendicular to the direction of propagation of the laser beam. The existence of vortex structures, obtained by monochromatic wave as a solution of the two-dimensional paraxial scalar equation of Leontovich was first presented by authors in [7]. The solutions admit amplitude and phase singularities. On the other hand, it is well-known that for these solutions the integral of energy is infinite. An elegant way to eliminate amplitude singularities is by using Gaussian pulses [8].

Important results in generating of optical vortices in fibers are presented in [9-12]. With regard to the distribution of the refractive index *n* of the waveguides in which these vortex structures are obtained, we can distinct gradient and stepped optical fibers. We are interested in the evolution of optical pulses in Kerr-type nonlinear dispersive media with spatial dependence of the refractive index. With $\vec{U}$ is presented the vector amplitude function, describing the pulse envelope. In this case, the refractive index of the media is of the kind [13-15]:

$$n = n_o(\omega) + S_g(x^2 + y^2) + n_2\left|\vec{U}\right|^2,$$ (1)

where $n_0(\omega)$ and n_2 are respectively the linear and nonlinear refractive index, characterizing the dispersion and nonlinearity of the medium. The second term $S_g(x^2 + y^2)$ in (1) gives the spatial dependence of the refractive index and S_g is a constant.

Depending on the sign of the constant S_g, fibers are divided in two types [16]:

1. $S_g < 0$ - optical waveguides that have this type of spatial dependence of the refractive index are known as gradient fibers. The linear refractive index is maximal on the fiber axis and decreases smoothly to its periphery (Fig. 1 (a)).

2. $S_g > 0$ - optical waveguides that have this type of spatial dependence of the refractive index are also gradient but concave. Their linear refractive index has a smaller value along the fiber axis and it rises smoothly to its periphery (Fig. 1 (b)).
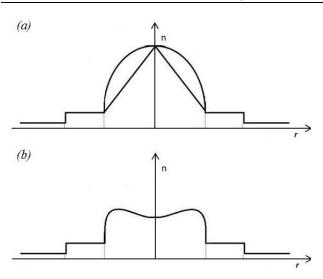
(a)

(b)

Fig. 1. Distribution of the refractive index n of gradient optical fibers; (a) gradient fibers; (b) concave gradient fibers

In this paper we are looking for vortex solutions of the nonlinear system of vector spatio-temporal amplitude equations of laser pulses, propagating in gradient optical fibers with spatial dependence of the refractive index.

2. Basic equations

The normalized nonlinear amplitude equation, describing the propagation of a linear polarized electrical field $\vec{U} = (U_x, U_y, 0)$ is of the kind [16]:

$$i\frac{\partial \vec{U}}{\partial z} + \frac{1}{2}\left(\Delta_\perp \vec{U} - S_d \frac{\partial^2 \vec{U}}{\partial t^2}\right) + S_g(x^2 + y^2)\vec{U} + \gamma|\vec{U}|^2\vec{U} = 0, \tag{2}$$

where $\Delta_\perp = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$.

Here, constant S_d characterizes the dispersion of the media. The parameter γ is nonlinearity of the medium and $\Delta_\perp$ is the transverse Laplace operator. This equation is a modified nonlinear Schrödinger equation, written in vector form in Cartesian coordinates. The additional third term describes the spatial dependence of the linear refractive index. An approximate solution of the equation above in scalar form for $S_g < 0$ is described in [16].

We are looking for an exact analytical solution of equation (2) in the case of $S_g > 0$. This condition applies to optical fibers and photonic crystals with a concave profile of the refractive index (Fig. 1*(b)*). Such types of waveguides find applications in different nonlinear devices for controlling and manipulating light, as optical transistors used in optical computers, as lenses and mirrors in thin-film optics, in modern optical sensors and communication systems [17-22].

We are looking for vortices solutions of the partial differential equation (2) for the vector amplitude function $\vec{U}(x, y, z, t) = (U_x(x, y, z, t), U_y(x, y, z, t), 0)$. In order to find them, we need to form the following system of scalar equations for each of the components U_x and U_y of the vector amplitude function:

$$\frac{\partial U_x}{\partial z} + \frac{1}{2}\left(\frac{\partial^2 U_x}{\partial x^2} + \frac{\partial^2 U_x}{\partial y^2}\right) - S_d \frac{\partial^2 U_x}{\partial t^2} + S_g(x^2 + y^2)U_x + \gamma|U_x^2 + U_y^2|U_x = 0, \tag{3}$$

$$\frac{\partial U_y}{\partial z} + \frac{1}{2}\left(\frac{\partial^2 U_y}{\partial x^2} + \frac{\partial^2 U_y}{\partial y^2}\right) - S_d \frac{\partial^2 U_y}{\partial t^2} + S_g(x^2 + y^2)U_y + \gamma|U_x^2 + U_y^2|U_y = 0, \tag{4}$$

where:
$$U_x = U_x(x, y, z, t),$$
$$U_y = U_y(x, y, z, t), \tag{5}$$

In order to find a solution of the system (3) and (4) we use the following mathematical algorithm:
- We will work in cylindrical coordinates:
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$$x = r\cos\theta, \quad y = r\sin\theta,$$

$$r = \sqrt{x^2 + y^2}, \quad \theta = \arctan(y/x), \tag{6}$$

$$\Delta_\perp A = \left(\frac{1}{r}\frac{\partial A}{\partial r} + \frac{\partial^2 A}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 A}{\partial \theta^2}\right).$$

- After these transformations, the system of equations (3) and (4) takes the form:

$$i\frac{\partial U_x}{\partial z} + \frac{1}{2}\left(\frac{1}{r}\frac{\partial U_x}{\partial r} + \frac{\partial^2 U_x}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 U_x}{\partial \theta^2}\right) - \frac{S_d}{2}\frac{\partial^2 U_x}{\partial t^2} + S_g r^2 U_x + \gamma|U_x^2 + U_y^2|U_x = 0, \tag{7}$$

$$i\frac{\partial U_y}{\partial z} + \frac{1}{2}\left(\frac{1}{r}\frac{\partial U_y}{\partial r} + \frac{\partial^2 U_y}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 U_y}{\partial \theta^2}\right) - \frac{S_d}{2}\frac{\partial^2 U_y}{\partial t^2} + S_g r^2 U_y + \gamma|U_x^2 + U_y^2|U_y = 0. \tag{8}$$

- We make the substitution:
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$$U_x = e^{i\psi_x(z,t)}P_x(r, t), \tag{9}$$
$$U_y = e^{i\psi_y(z,t)}P_y(r, t),$$

where P_x and P_y are initial peak powers of the amplitude functions of the pulses and ψ_x and ψ_y are the phases of the components.

- Having in mind that U_x and U_y are components of the same pulse, we assume that the phase functions are the same:

$$\psi_x = \psi_y = \Psi(z,t). \qquad (10)$$

Then:

$$U_x = P_x\,(r,t)e^{i\Psi(z,t)},$$
$$U_y = P_y\,(r,t)e^{i\Psi(z,t)}. \qquad (11)$$

Thus, after a couple of transformations the system of equations (7) and (8) takes the form:

$$-\frac{\partial \Psi}{\partial z}P_x\,e^{i\Psi} + \frac{1}{2}\left(\frac{1}{r}\frac{\partial P_x}{\partial r} + \frac{\partial^2 P_x}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 P_x}{\partial \theta^2}\right)e^{i\Psi} - \frac{S_d}{2}\left(i\frac{\partial^2 \Psi}{\partial t^2} - \left(i\frac{\partial \Psi}{\partial t}\right)^2\right)P_x e^{i\Psi} + S_g r^2 P_x e^{i\Psi}$$
$$+ \quad \gamma\left|P_x^2(r,\theta) + P_y^2(r,\theta)\right|P_x e^{i\Psi} = 0,$$

$$(12)$$

$$-\frac{\partial \Psi}{\partial z}P_y\,e^{i\Psi} + \frac{1}{2}\left(\frac{1}{r}\frac{\partial P_y}{\partial r} + \frac{\partial^2 P_y}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 P_y}{\partial \theta^2}\right)e^{i\Psi} - \frac{S_d}{2}\left(i\frac{\partial^2 \Psi}{\partial t^2} - \left(i\frac{\partial \Psi}{\partial t}\right)^2\right)P_y e^{i\Psi} + S_g r^2 P_y e^{i\Psi}$$
$$+ \quad \gamma\left|P_x^2(r,\theta) + P_y^2(r,\theta)\right|P_y e^{i\Psi} = 0.$$

$$(13)$$

- We divide the two sides of the system above of $P_x\,e^{i\Psi(z,t)}$ and $P_y\,e^{i\Psi(z,t)}$. As a result the equations (12) and (13) take the form:

$$\left[\left(\frac{\partial \Psi}{\partial z} + i\frac{S_d}{2}\frac{\partial^2 \Psi}{\partial t^2} - i\frac{S_d}{2}\left(\frac{\partial \Psi}{\partial t}\right)^2\right)\right] = \frac{1}{2}\left(\frac{1}{r}\frac{\partial P_x}{\partial r} + \frac{\partial^2 P_x}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 P_x}{\partial \theta^2}\right)\frac{1}{P_x} + S_g r^2 + \gamma\left|P_x^2(r,\theta) + P_y^2(r,\theta)\right|, \qquad (14)$$

$$\left[\left(\frac{\partial \Psi}{\partial z} + i\frac{S_d}{2}\frac{\partial^2 \Psi}{\partial t^2} - i\frac{S_d}{2}\left(\frac{\partial \Psi}{\partial t}\right)^2\right)\right] = \frac{1}{2}\left(\frac{1}{r}\frac{\partial P_y}{\partial r} + \frac{\partial^2 P_y}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 P_y}{\partial \theta^2}\right)\frac{1}{P_y} + S_g r^2 + \gamma\left|P_x^2(r,\theta) + P_y^2(r,\theta)\right|. \qquad (15)$$

- After that we divide the variables in the two equations. In the system above, the left terms of the equations are not functions of r and the right terms are not functions of z and t. Therefore, in order the equalities to be fulfilled, it is necessary that each of the sides of the equations to be equal to a constant. So the following systems of equations are obtained:

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$$\frac{\partial \Psi}{\partial z} + i\frac{S_d}{2}\frac{\partial^2 \Psi}{\partial t^2} - i\frac{S_d}{2}\left(\frac{\partial \Psi}{\partial t}\right)^2 = a = const, \qquad (16)$$

$$\frac{1}{2}\left(\frac{1}{r}\frac{\partial P_x}{\partial r} + \frac{\partial^2 P_x}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 P_x}{\partial \theta^2}\right)\frac{1}{P_x} + S_g r^2 +$$
$$\gamma\left|P_x^2(r,\theta) + P_y^2(r,\theta)\right| = a = const, \qquad (17)$$

$$\frac{\partial \Psi}{\partial z} + i\frac{S_d}{2}\frac{\partial^2 \Psi}{\partial t^2} - i\frac{S_d}{2}\left(\frac{\partial \Psi}{\partial t}\right)^2 = d = const, \qquad (18)$$

$$\frac{1}{2}\left(\frac{1}{r}\frac{\partial P_y}{\partial r} + \frac{\partial^2 P_y}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 P_y}{\partial \theta^2}\right)\frac{1}{P_y} + S_g r^2 +$$
$$\gamma\left|P_x^2(r,\theta) + P_y^2(r,\theta)\right| = d = const. \qquad (19)$$

The equations (16) and (18) are referred to the same function, so $a = d$, i.e. the both equations coincide.

- Let's first take into account the equation (16). We make the following substitution for the phase function:

$$\Psi = bt + cz, \qquad (20)$$

where $\quad \frac{\partial \Psi}{\partial z} = c, \quad \frac{\partial \Psi}{\partial z} = b.$

After we substitute the expressions above in equation (16), we obtain:

$$c - \frac{S_d}{2}b^2 = a\,. \qquad (21)$$

The equality above presents the connection between the constants a, b and c.

- Now, we will consider the equations (17) и (19). They are of the same type and are referred to the same unknown functions P_x and P_y. We will find their solutions by making the substitutions:

$$P_x = \frac{A}{r}e^{igr^2}T_x(\theta), \quad P_y = \frac{A}{r}e^{igr^2}T_y(\theta), \qquad (22)$$

where T_x and T_y are new unknown functions of the variables z, t and θ that characterize the initial amplitude functions. The angle θ is connected to the rotation of the vector of the electrical field and A and g are constants.

As a result the following equations for the functions T_x and T_y are obtained:

$$\frac{1}{2}\left(\frac{1}{r^3} - 4g^2 r\right)r + S_g r^2 + \frac{1}{2r^2}\frac{\partial^2 T_x}{\partial \theta^2}\frac{1}{T_x} + \gamma\frac{A^2}{r^2}\left|T_x^2 + T_y^2\right| = a, \qquad (23)$$

$$\frac{1}{2}\left(\frac{1}{r^3} - 4g^2r\right)r + S_g r^2 + \frac{1}{2r^2}\frac{\partial^2 T_y}{\partial \theta^2}\frac{1}{T_y} + \gamma\frac{A^2}{r^2}\left|T_x^2 + T_y^2\right| = a \qquad (24)$$

On the right side of the equations above, it is presented the sum of the squares of the functions T_x and T_y. That is why it is appropriate to wright down:

$$T_x = cos(k\theta), \qquad (25)$$

$$T_y = sin(k\theta). \qquad (26)$$

where k is a constant.

Thus, we replace (25) and (26) in the system of equations (23) and (24). After a couple of transformation the system is reduced to one equation:

$$\frac{1}{2r^2}(1 - k^2 + 2\gamma A^2) + r^2(S_g - 2g^2) = a. \qquad (27)$$

In the expression above the first term has a multiplier r^{-2}, and the second one respectively r^2. In order the equality to be satisfied it is necessary that:

$$a = 0, \qquad (28)$$

$$(1 - k^2 + 2\gamma A^2) = 0, \qquad (29)$$

$$(S_g - 2g^2) = 0. \qquad (30)$$

By using the nonlinear dispersion relation (29) we can define the constant A

$$A = \sqrt{\frac{k^2-1}{2\gamma}}, \qquad (31)$$

and the connection between the parameters

$$g = \pm\sqrt{\frac{S_g}{2}} \ . \qquad (32)$$

Having in mind (21), (32) and $a = 0$, we find a link between the constants c and b:

$$c = \frac{S_g}{2}b^2. \qquad (33)$$

when $b = 1 \ \Rightarrow c = \frac{S_g}{2}$.

Thus, by using the substitutions above, we found exact analytical solutions of the nonlinear system of equations (3) and (4).

$$U_x = \sqrt{\frac{k^2-1}{2\gamma}}\frac{1}{r}e^{i\sqrt{\frac{S_g}{2}}r^2}cos(k\theta)e^{i(t+\frac{S_d}{2}z)}, \qquad (34)$$

$$U_y = \sqrt{\frac{k^2-1}{2\gamma}}\frac{1}{r}e^{i\sqrt{\frac{S_g}{2}}r^2}sin(k\theta)e^{i(t+\frac{S_d}{2}z)}, \qquad (35)$$

where k is the vortex number, $k \neq 1 , k = 2, 3, 4, 5 \ldots$

The two expressions present the amplitude function of the components of the vector electrical field $\vec{U}$. The number 1 in the nonlinear dispersion relation (29) comes from the diffraction terms. The parameter k^2 is obtained from the angular terms and the angular distribution. It is obvious that when $k=0$ there are no vortices and there are no solutions of the basic system of equations, because A^2 is positive. The vortex solutions (34) and (35) are solutions of the system of equations (3) and (4) only if $r \neq 0$ and the nonlinear dispersion relation (29) is satisfied. This corresponds to the fact that the obtained vortices have discrete series of the amplitudes. Usually, in linear theory of optical vortices the peak amplitude constants have arbitrary values, because they do not present in the linear dispersion relation. In the nonlinear dispersion relations of these new solutions the amplitude constants are presented.

3. Numerical calculations

We have made a couple of simulations for the analytical solutions (34) and (35) of the system of equations (3) and (4) for different values of the vortex number k. They characterize the pulse envelope, propagating in nonlinear dispersive fiber with spatial dependence of the refractive index.

In numerical experiments vortices are usually investigated by using masks in the center of vortex structure. That corresponds to adding an additional term ($\alpha<<1$) in the denominator of the solutions.

• First numerical simulations are made for $k=2$. In this case the exact analytical solutions (34) and (35) of the basic system of equations are of the kind:

$$U_x = \sqrt{\frac{3}{2}}\frac{1}{r}e^{i\sqrt{\frac{S_g}{2}}r^2}cos(2\theta)e^{i(t+\frac{S_d}{2}z)}, \qquad (36)$$

$$U_y = \sqrt{\frac{3}{2}}\frac{1}{r}e^{i\sqrt{\frac{S_g}{2}}r^2}sin(2\theta)e^{i(t+\frac{S_d}{2}z)}. \qquad (37)$$

The solutions above can be written in the Cartesian coordinates:

$$U_x = \sqrt{\frac{3}{2}}e^{i\sqrt{\frac{S_g}{2}}r^2}\frac{x^2-y^2}{r^3}e^{i(t+\frac{S_d}{2}z)}, \qquad (38)$$

$$U_y = \sqrt{\frac{3}{2}}e^{i\sqrt{\frac{S_g}{2}}r^2}\frac{2xy}{r^3}e^{i(t+\frac{S_d}{2}z)}. \qquad (39)$$

In Fig. 2 (a) and (b) there are presented the profiles of the components U_x and U_y of the amplitude function of the optical vortex. It is quite complicated but symmetrical distribution of the intensity of the pulse.
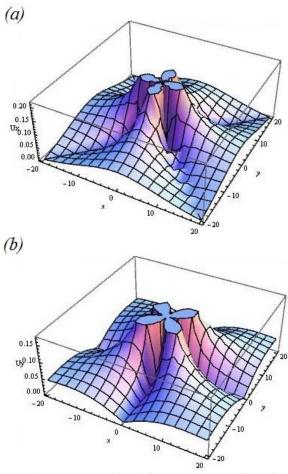
(a)

(b)

Fig. 2. Intensity profiles of the components (a) U_x and (b) U_y in the case of k=2

In Fig. 3 it is presented the intensity profile of the vortex structure. It is observed a symmetrical distribution of the vortex energy in the pedestal.
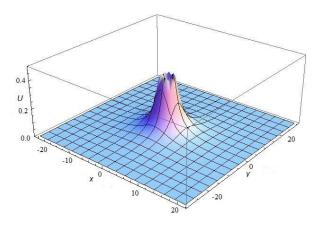

Fig. 3. Intensity profile of the optical vortex in the case of k=2

On Fig. 4 it is shown the diagram of the vector amplitude function of the optical vortex in the case of *k=2*. It is observed a rotation of the vector of the electrical field in the center of the vortex.
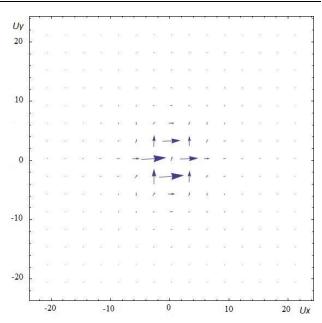

Fig. 4. Diagram of the vector amplitude function for k=2. Significant rotation of the vector of the electrical field in the center of the vortex is observed

- For *k=3* the solutions of the system of equations (3) and (4) are of the kind:

$$U_x = \sqrt{2}\frac{1}{r}e^{i\sqrt{\frac{S_g}{2}}r^2}cos(3\theta)e^{i(t+\frac{S_d}{2}z)}, \tag{40}$$

$$U_y = \sqrt{2}\frac{1}{r}e^{i\sqrt{\frac{S_g}{2}}r^2}sin(3\theta)e^{i(t+\frac{S_d}{2}z)}. \tag{41}$$

These solutions, written in Cartesian coordinates are in the form:

$$U_x = \sqrt{2}e^{i\sqrt{\frac{S_g}{2}}r^2}\frac{4x^3-3xr^2}{r^4}e^{i(t+\frac{S_d}{2}z)}, \tag{42}$$

$$U_y = \sqrt{2}e^{i\sqrt{\frac{S_g}{2}}r^2}\frac{3yr^2-4y^3}{r^4}e^{i(t+\frac{S_d}{2}z)}. \tag{43}$$

In Fig. 5 *(a)* and *(b)* there are presented the profiles of the components U_x and U_y of the amplitude function of the optical vortex for *k=3*. Here, the distribution of the intensity of the pulse is even more complicated but also symmetrical.
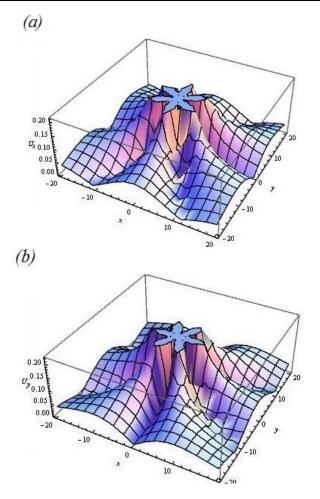
On Fig. 7 it is shown the diagram of the vector amplitude function of the optical vortex in the case of $k=3$. Here, it is also observed a significant rotation of the vector of the electrical field in the center of the vortex.

(a)

(b)

Fig. 5. Intensity profiles of the components (a) U_x and (b) U_y in the case of $k=3$

In Fig. 6 it is presented the profile of the intensity of the vortex structure in the case of $k=3$. It is observed the same pedestal as that on Fig. 3.
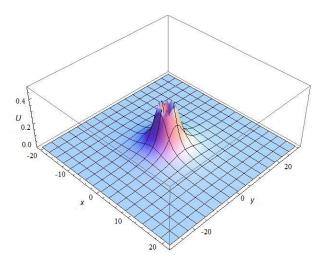

Fig. 6. Intensity profile of the optical vortex in the case of $k=3$
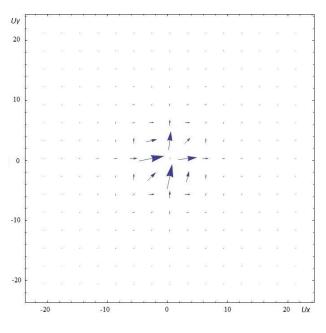

Fig. 7. Diagram of the vector amplitude function for $k=3$. Significant rotation of the vector of the electrical field in the center of the vortex is observed

4. Conclusions

In the present work it was shown a mathematical algorithm for solving the nonlinear system of spatio-temporal amplitude equations (3) and (4), describing the propagation of the components of laser pulses in isotropic nonlinear dispersive optical fiber with spatial dependence of the refractive index when $S_g > 0$.

A new class of exact analytical solutions (34) and (35) in the form of optical vortex structures are found. The nonlinear dispersion ratios obtained by these vortex solutions shows that their stability is due to the balance between diffraction and nonlinearity, as well as the balance between nonlinearity and angular distribution of the field. A number of numerical simulations of the solutions of the system of equations (3) and (4) are made.

Acknowledgments

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