Wave packet dynamics for a system in the quantum well with one moving wall

DE-HUA WANG[a,*], HUA WANG[b], HAI-FENG MU[a], TONG SHI[a], XIN-YUE SUN[a], Gang ZHAO[a], QIN-FENG XU[a]
[a]*School of Physics and Optoelectronic Engineering, Ludong University, Yantai 264025, China;*
[b]*Department of Electronic Engineering, Yantai Automobile Engineering Professional College, Yantai 265500, China*

The time evolution of the wave function provides a beautiful insight into the wave packet dynamics in the quantum theory. In this work, we study the wave packet dynamics of a particle inside a quantum well with one moving wall. Since one wall is moving with time, the eigen wave function and eigen energy of this system are time-dependent, which makes the time evolution of the wave packet in this system become much more complicated and interesting compared to the case in a quantum well with two static walls. It is shown when the wall is moving very slowly, fractional revival phenomenon of wave packet is shown to exist, but the full revival of the wave packet at one revival time is disappeared. With the increase of the moving speed of one wall, a peculiar revival pattern occurs, the center of the revival peak will shift accompanied by the motion of the wall, the wave function can not "clone" its initial wave function at all. Revivals are observed at some special decimal fractions of the revival time instead of rational fractions of the revival time appeared in the case of the static quantum well. Additionally, the initial momentum of the Gaussians wave packet can also affect the wave packet evolution and the autocorrelation of this system. The probability density distribution and the autocorrelation of this system are nearly unaffected for Guassian with an initial momentum smaller than the wall motion. However, when the initial momentum of the Gaussians wave packet is larger than the wall motion, the amplitude in the probability density gets decreased, but the oscillatory structures become complicated; the interval between the adjacent revival peaks in the autocorrelation function decreases with the initial momentum. The non-periodic revival pattern observed in this work can be considered as a consequence of the moving wall effect. We hope that our work can guide the future experimental study of the wave packet dynamics in the presence of a moving boundary.

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1. Introduction

Over the last several decades, the wave packet revival phenomenon in quantum systems has attracted a great interest. In the theoretical aspect, the temporal evolution of quantum wave packet has been studied using a pump-probe method of detection involving either time-delayed photoionization [1] or phase modulation [2-4], which has been widely used in atomic and nonlinear quantum systems [5-7]. Revivals in quantum systems were first studied in the Jaynes-Cummings model, describing a two-level atom interacting with a resonant monochromatic field [8]. On the other hand, the phenomena of wave packet revivals have been observed in many experimental studies, such as in Rydberg wave packets for hydrogen and alkali-metal atoms [9-12], in molecular vibrational state wave packet for Na_2 and Br_2 [13-14], etc. On the basis of the wave packet revival phenomena, some methods for isotope separation [15], and wave packet control [16-18] have been put forward. Systems exhibiting revival behavior are a fundamental realization of time-dependent interference phenomena for bound states with quantized energies in quantum mechanics and have wide applications in the field of physics and chemistry [19]. Application of wave packets to analyze the dynamical property of quantum systems is an important method to study the correspondence between the classical and quantum mechanics [20].

Among all studies, the infinite square well potential is an elementary and important model to study the time evolution of the wave packet in quantum mechanics, because it provides a good description for one-dimensional bound state problem, such as in the semiconductor quantum well or in the billiard systems [21,19]. In these systems, the confined potential acting on the particle can be considered as an infinite square well. For the infinite square well system, the wave function and energy have simple analytic expressions, which can be obtained easily by solving the time-independent Schrödinger equation. In such system, the phenomena of full and part revival in the wave packet are well known. Such phenomena play a significant role in our understanding of wave packet evolution. In the previous studies, many researchers have studied the time evolution of wave function in this system. For example, in 1997, Aronstein et al have studied the time evolution of a wave function in the infinite well using a fractional revival formalism [21]. They found a fractional revival of wave function occurs at times equal to rational fractions of the revival time T_{rev}. At one revival time, the wave function can exactly reproduce its initial form, which is called a full revival. Later, Schmidt investigated the wave packet revival for a one-dimensional system with position-dependent mass inside an infinite well [22]. They found approximate full and fractional revivals in the wave packet, and the full revival takes place faster than the usual case. Furthermore, Vubangsi and their group have examined the wave packet revival dynamics for a quantum system with position and time-dependent mass in an infinite well [23]. Revival of wave-packet is shown to exist and partial revivals are different from the case with constant mass.

In these early studies, the researchers all studied the wave packet dynamics in the infinite well with two static walls. Then what will happen for the wave packet dynamics of a quantum system in the infinite well with one moving wall? For the moving boundary problem, the temporal evolution of the wave packet becomes complicated. However, the quantum systems with time-dependent boundary conditions induced by the moving wall have attracted a lot of attention for several decades [24-37]. For example, in the early 1969, Doescher and rice investigated the system of a particle in a one-dimensional infinite square well potential with one expanding or contracting wall [24]. Canon derived the one-dimensional heat equation considering the moving boundary condition[25].Pinder has studied the contracting square well and analyzed the reason why the sudden approximation is inapplicability for contracting walls[26]. Scheininger and Kleber studied the quantum to classical correspondence for a particle confined in a quantum well with a linear wall motion and showed that a slight change in the boundary condition would lead to chaotic motion [27].Furthermore, Luz and Cheng used the semiclassical approximation to evaluate the propagators for moving hard-wall potentials [28]. Aslangul studied the time evolution of a particle in the sudden-expanded infinite well [29].Glasser et al have studied the time-dependent wave function for the quantum infinite square well with an oscillating wall [30]. Fojón's group have used a numerical analysis to study the quantum square well with moving boundaries [31]. Mousavi investigated the quantum effective force in an expanding square-well potential [32]. Maritino et al studied the problem of a quantum particle that bounces back and forth between two moving walls [33]. Cooney studied the evolution of a wavefunction in an infinite well with moving walls. They derived the location and time of the revivals in the well of the initial wavefunction using Jacobi's elliptic theta function [34]. As to the evolution and revival of the wave packet from the aspect of the autocorrelation, he did not give a discussion. Very recently, Duffin et al have numerically studied a particle in a box with oscillating walls[35]; Matzkin and their coworkers investigated the issue of a single particle nonlocality in a quantum system subjected to time-dependent boundary conditions[36,37]. In the above studies, the energy and wave function of a particle in an infinite well with one wall moving with a constant velocity have been put forward.

Based upon these studies, we study the wave packet dynamics for a system with one moving wall in the quantum well. For simplicity, we still consider the walls of the quantum well are infinitely high. If one wall is moving, the width of the well grows wider, or narrower. Accordingly, the wave function of this system will grow or shrink and the energy of the particle will increase or decrease with time. In order to study the wave packet dynamics of this system, we should expand the initial wave function at arbitrary time in terms of these wave functions. The moving wall will affect the behavior of the wave packet in the quantum well through the following two aspects. First, if one wall is moving with a constant velocity, the value of the energy E_n is time-dependent, and the term corresponding to $\exp[-i\int_0^t E_n d\tau]$ will produce an extra dynamical phase shift in contrast to the case in the static quantum well. Second, the center of the probability density corresponding to each wave function will shift accompanied by the motion of the wall. The above two effects will all influence the time evolution and revival of the wave packet in the quantum well. It is found that the temporal evolution of the wave packet in this system is quite different from the case in a quantum well with two static walls, which is determined by the moving speed of the wall. It has been noted that as one wall is moving very slowly, the initial wave packet can be re-constructed partially after a period of time, but the full revival of the wave packet is disappeared. When the wall is moving very fast, the mirror revival or the fractional revival occurred at one-half and one-quarter fractions of revival times are no longer exist. Instead, revivals are observed for some special decimal fractions of the revival time. These non-periodic revival patterns are induced by the moving wall. This work can provide some references for the future experimental study of the wave packet dynamics in the presence of a moving boundary.

2. The wave function and classical motion of a particle in the quantum well with one moving wall

2.1. The wave function

Suppose a particle is confined in a quantum well with one moving wall. One wall in the well is fixed at origin and the other wall is moving with a constant velocity v along the x axis. At first, the width of the well is l_0, at time t, its width becomes l: $l=l_0+vt$, where v can be positive or negative. If $v>0$, the wall is moving rightward, which makes the quantum well get wider; however, if $v<0$, the wall is moving leftward, and the quantum well becomes narrower.

The potential acting on the particle in this quantum well can be described as:

$$V(x,t)=\begin{cases}0 & 0\le x\le l\\ +\infty & x>l\end{cases} \qquad (1)$$

By solving the time-dependent Schrödinger equation:

$$i\hbar\frac{\partial\psi(x,t)}{\partial t}=\{-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}+V(x,t)\}\psi(x,t) \qquad (2)$$

with the boundary conditions $\psi(x=0)=\psi(x=l)=0$, we obtain the wave function of this system[38]:

$$\psi_n(x,t)=\sqrt{\frac{2}{l}}\exp[i\frac{mvx^2}{2\hbar l}]\sin[\frac{n\pi x}{l}] \times\exp[-i\int_0^t E_n d\tau] \qquad (3)$$

where E_n is the energy: $E_n=\frac{n^2\pi^2}{2ml^2}$. In contrast to the system in the static quantum well, the wave function and energy all vary with time. In the following calculation, we choose the natural units: $m=\hbar=1$.

These wave functions possess a sort of orthogonal normalized quality with the condition:

$$\int_0^l dx\psi_n^*(x,t)\psi_n(x,t)=\delta_{nm} \qquad (4)$$

In order to study the time evolution of the wave packet in the quantum well with one moving wall, we choose the initial Gaussian wave packet as follows:

$$\psi(x,0)=\sqrt{\frac{1}{\sigma\sqrt{2\pi}}}\exp[iP_0(x-x_0)-\frac{(x-x_0)^2}{4\sigma^2}] \qquad (5)$$

where P_0 is the initial momentum, x_0 specifies the mean position of the wave packet, and σ determines the spread of the wave packet. In the following calculation, we choose $x_0=20$ and $\sigma=2$. $\psi(x,0)$ can be expanded using the eigen wave function $\psi_n(x,0)$:

$$\psi(x,0)=\sum_n C_n\psi_n(x,0) \qquad (6)$$

Here C_n is the expansion constant: $C_n=\int_0^{l_0}dx\,\psi_n^*(x,0)\psi(x,0)$. C_n gives the projection of the initial wave function on the nth energy eigenstate. According to the quantum mechanics, $|C_n|^2$ is the probability of measuring the system to appear in that eigenstate.

At arbitrary time t, the wave function can be written as:

$$\begin{aligned}\psi(x,t)&=\sum_n c_n\psi_n(x,t)\\ &=\sqrt{\frac{2}{l}}\exp[i\frac{mvx^2}{2\hbar l}]\sum_n c_n\sin[\frac{n\pi x}{l}]\\ &\times\exp[-i\int_0^t E_n d\tau]\end{aligned} \qquad (7)$$

A widely used method for probing the evolution and revival of the wave packet dynamics is based on the autocorrelation function [39], which is directly related to the observable signal in the pump-probe type experiments for studying the wave packet dynamics. The autocorrelation function measures the overlap of the initial state wave packet $\psi(x,0)$ with the state $\psi(x,t)$ at some later time, which is defined as:

$$\begin{aligned}A(t)&=<\psi(x,t)|\psi(x,0)>\\ &=\sum_n|c_n|^2\exp[i\int_0^t E_n d\tau]\end{aligned} \qquad (8)$$

Using the autocorrelation function, the occurrence of revival and fractional revivals in the wave packet corresponds to the value of $A(t)$ to its initial value of unity and the appearance of relative maxima in $A(t)$, respectively.

It has been demonstrated in many researches that the important time scales of a wave function's evolution are contained in the coefficients of the Taylor series of the energy levels E_n around the mean energy n_0[19]:

$$E_n = E_{n_0} + E'_{n_0}(n-n_0) + \frac{1}{2!}E''_{n_0}(n-n_0)^2 + \frac{1}{3!}E'''_{n_0}(n-n_0)^3 + ... \quad (9)$$

where $E'_{n_0} = (dE_n/dn)_{n=n_0}$ and so forth. For this system, only E'_{n_0} and E''_{n_0} are unequal to zero, so we retain the first three terms. The first term E_{n_0} is a constant and induces no interference between different wave function, and it has no observable effect in $|\psi(x,t)|^2$ and $|A(t)|^2$, so this term can be neglected. Therefore, we define two time scales that depend on E'_{n_0} and E''_{n_0}. One is T_{cl}, which is associated with the classical period of motion in the bound state. Another is the revival period T_{rev}, which governs the revival and fractional revival of the wave packet:

$$T_{cl} = \frac{2\pi}{|E'_{n_0}|} = \frac{2l^2}{n_0\pi} \quad (10)$$

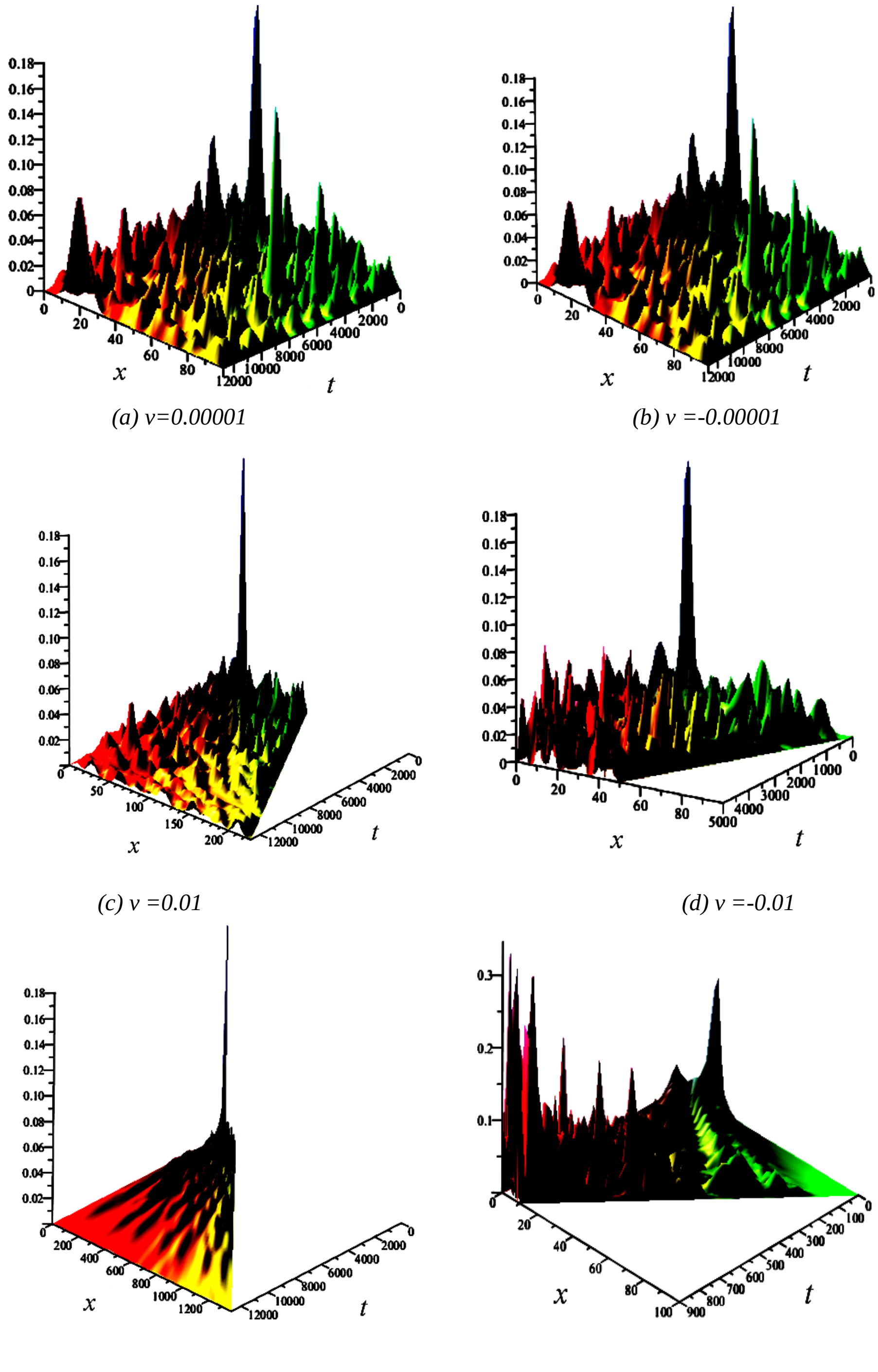
$$T_{rev} = \frac{4\pi}{|E''_{n_0}|} = \frac{4l^2}{\pi} = 2n_0 T_{cl} \quad (11)$$

2.2. The classical motion of a particle in the quantum well with one moving wall

The classical motion of this system can be described as follows: Suppose the particle initially lies at the origin and travels with a constant speed k. If we take the particle's energy to be $E = E_{n_0} = \frac{n_0^2\pi^2}{2l_0{}^2}$, then the speed of the particle $k = \frac{n_0\pi}{l_0}$. In the following calculation, we choose $n_0 = 15$. The particle moves freely in the quantum well, with the trajectories along straight lines inside the quantum well until they are reflected by the walls. After one reflection by the moving wall, the particle may return to the origin to form a closed orbit. Assuming the mass of the moving wall is far larger than that of the particle and the collision between the particle and the wall is completely elastic. The period of the particle in the quantum well depends on the moving speed and direction of the right wall. We take the wall moves rightward as an example. If the initial speed of the particle k is larger than the speed of the moving wall v, the particle will hit the moving wall after some time. This period of time is denoted as T_1: $T_1 = \frac{l_0}{k-v}$. After collision with the right wall, the returning speed of the particle becomes: $k_{ret}=k-2v$. The returning time it takes the particle from the moving wall to the origin is T_2: $T_2 = \frac{k}{k-2v}T_1$. Therefore, the first round-trip period of the particle along the x axis is: $T_{rt}^{(1)} = T_1 + T_2 = \frac{2l_0}{k_0 - 2v}$. After collision with the left wall, it turns back and begins the second round trip. If we use "j" as the number of collisions with the moving wall, then the j-th round-trip time of the particle in the quantum well can be described as: $T_{rt}^{(j)} = \frac{2jl_0}{k_0 - 2jv}$. If the moving wall moves leftward at first, we can use the same method to describe the motion of the particle. Such description of a classical particle has been proven useful for constructing exact propagators for path-integral solutions to infinite square-well system with one moving wall [24].

3. The evolution and revival of the wave packet in the quantum well with one moving wall

In Fig. 1, we plot the three dimensional probability density $|\psi(x,t)|^2 = |\sum_n c_n\psi_n(x,t)|^2$ in the quantum well with one moving wall. Suppose the initial momentum of the Gaussian wave packet $P_0 = 0$. The left column corresponds to the case with the wall moving rightward, and the right column is the case with the wall moving leftward. It is shown when the wall is moving very slowly, v=0.00001, no matter the wall is moving rightward or leftward, the difference in the probability density distribution is very small, as we show in Fig. 1 (a-b). With the increase of the moving speed, the difference becomes obvious. When the wall is moving rightward, the probability density distribution region becomes wider, and the value of the probability density gets decreased.



(a) v=0.00001 *(b) v =-0.00001*

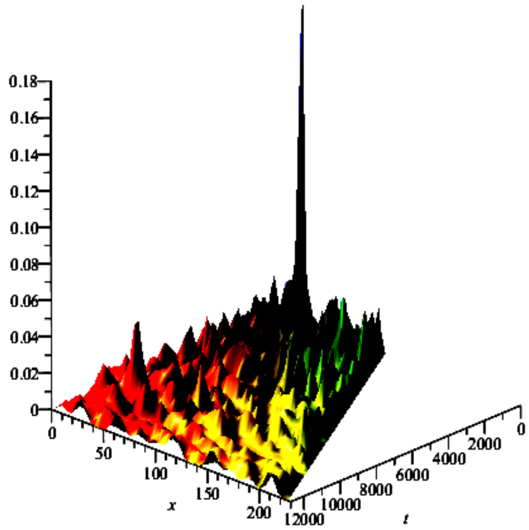
(c) v =0.01 *(d) v =-0.01*

(e) v =0.1 *(f) v =-0.1*

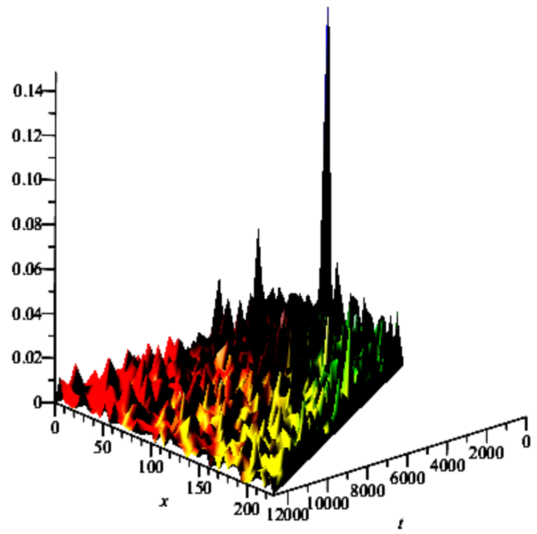
Fig. 1. (color online) The three dimensional probability density $|\psi(x,t)|^2$ *versus x and t in the quantum well with one moving wall. The left column corresponds to the case with the wall moving rightward, and the right column is the case with the wall moving leftward. Suppose the initial momentum of the Gaussian wave packet* P_0 *=0. The moving speed of the wall is given in each plot*

As v=0.1, the probability density distribution is expanded in a large region and looks like a peacock spreading its tail (see Fig. 1 (e)). In contrast, when the wall is moving leftward, the probability density distribution region becomes narrower, and the value of the probability density gets increased. For example, in Fig. 1 (f), the probability density is limited only in a small region $0<x<10$, and its value is very large.

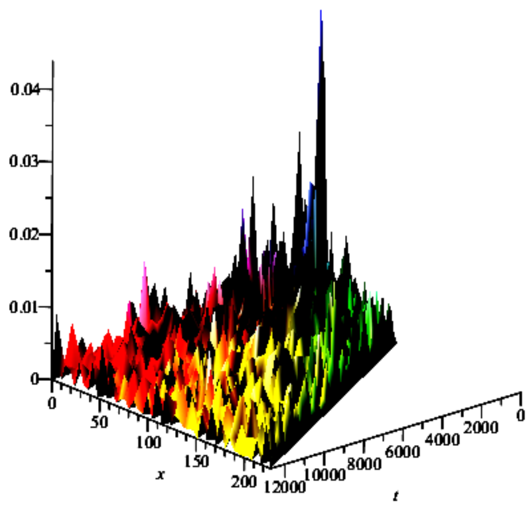
We next show in Fig. 2 the effect of the initial momentum in the Gaussian wave packet on the probability density distribution in the quantum well with one moving wall. Suppose the wall is moving with a speed v=0.01. Fig. 2 (a) shows the case with the initial momentum is less than the moving speed of the wall, P_0=0.005, the probability density distribution is nearly unchanged compared with Fig.1(c), which is the case with zero momentum. As we increase the initial momentum, $P_0 > v$, its effect begins to take place. Fig. 2 (b) shows the probability density distribution with P_0 =0.5, we found the value of the maximum recurrence peak gets decreased. As we further increase the initial momentum, the value of the recurrence peak continues to decrease, and the oscillatory structures in the probability density distribution gets much more complicated, as we can see from Fig. 2 (c-d).



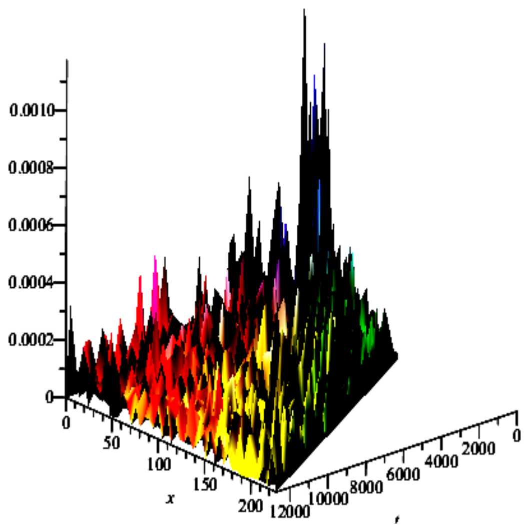
(a) P_0 =0.005



(b) P_0 =0. 5



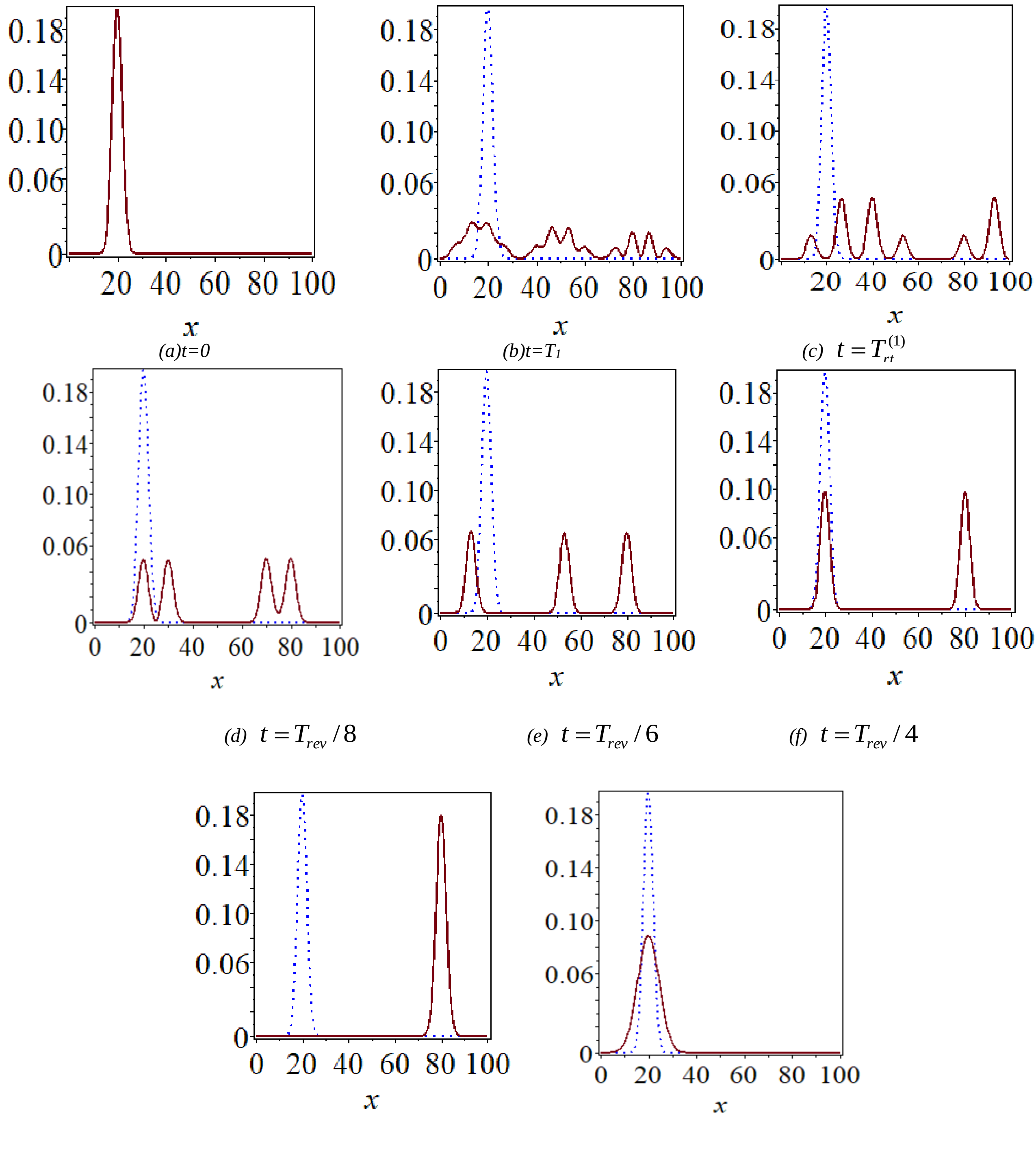
(c) P_0 =1.0



(d) P_0 =1.5

Fig. 2. (color online) The effect of the initial momentum in the Gaussian wave packet on the dimensional probability density in the quantum well with one moving wall. Suppose the wall is moving rightward with a speed v=0.01.The initial momentum of the Gaussian wave packet is given in each plot

In order to see the time evolution of the wave packet in the quantum well very clearly, we plot the two-dimensional probability density distribution at different time. Figs. 3 and 4 show the probability density distribution with the wall moving rightward for different velocity. In Fig. 3, the wall moves very slowly, v=0.00001. Fig. 3 (a) shows the probability density distribution of the initial wave packet, a sharp peak is centered at $x=x_0=20$. The probability density distribution in the quantum well varies as time goes on. The blue dotted line in each subplot denotes the probability density of the initial wave packet, which is given for comparison.



(a) $t=0$ (b) $t=T_1$ (c) $t=T_{rt}^{(1)}$

(d) $t=T_{rev}/8$ (e) $t=T_{rev}/6$ (f) $t=T_{rev}/4$

(g) $t=T_{rev}/2$ (h) $t=T_{rev}$

Fig. 3. (color online) The probability density distribution $|\psi(x,t)|^2$ versus x in the quantum well with one wall moving very slowly. The moving speed of the wall is v=0.00001.The times are given in each plot

Fig. 3 (b) shows the probability density distribution at $t = T_1 = \frac{l_0}{k - v} = 212.21$. At this time, the particle hits the moving wall for the first time. It is shown the wave packet is expanded in the whole well. Several peaks appear in the probability density distribution. At the first round-trip period of the particle in the quantum well, $t = T_{rt}^{(1)} = \frac{2l_0}{k_0 - 2v} = 424.41$(Fig. 3 (c)), the adjacent peaks in Fig. 3 (b) are separated into distinct peaks. At $t = T_{rev}/8 = 1591.55$, where $T_{rev} = 4l_0^2/\pi$ is the revival period of the particle in the quantum well with two static walls, fractional revival of the wave packet appears, the position of the first peak is consistent with the initial wave packet, but its height gets reduced, as we can see from Fig. 3 (d).

Fig. 3 (e) shows the probability density distribution at $t = T_{rev}/6$, we find three identical peaks centered at *x*=10,50 and 80, respectively. The first peak deviates from the initial wave packet. At $t = T_{rev}/4$, we observe two asymmetrical peaks in the probability density distribution, fractional revival of the wave packet appears again. As time continues, at $t = T_{rev}/2$, there is a single reflected copy of the initial wave packet centered at *x*= 80, which can be considered as a mirror revival of the wave packet. Finally, at one revival period, $t = T_{rev}$, there is only one peak localized at *x*= 20, but its width is a little wider and its height is lower than the initial wave packet. The exact full wave packet revival does not appear in contrast to the case in the static quantum well [19].

Fig. 4 shows the probability density distribution with one wall moves a little fast, *v*=0.01. Fig. 4 (a) shows the probability density distribution when the particle hits the moving wall for the first time, i.e. at $t = T_1 = \frac{l_0}{k - v} = 216.81$. At this time, the wave packet begins to collapse. At the first round-trip period of the particle in the quantum well $t = T_{rt}^{(1)} = \frac{2l_0}{k_0 - 2v} = 443.22$, several distinct peaks appear, but none peak is consistent with the initial wave packet. With the increase of time, the probability density distribution region in the quantum well gets enlarged. At $t = T_{rev}/8$, $t = T_{rev}/4$ and $t = T_{rev}/2$, the fractional wave packet revival, symmetry revival and the mirror revival phenomena appeared in Fig.4 are no longer exist. At one revival period, $t = T_{rev}$, we still could not observe a partial revival, the wave packet is collapsed completely. As we can see from Fig. 4 (f) clearly. However, if we further subdivide the evolution time, as shown in Fig. 4 (g-*l*), we find some distinct revival peaks in the probability density distribution, and the first peak moves rightward relative to the initial wave packet. For example, in Fig. 4 (*g*), $t = 0.149T_{rev} = 1897$, the center of the first peak is moved to *x*=24. With the increase of time, the first peak continues moving rightward. In Fig. 4 (*l*), $t = 0.815T_{rev} = 10377$, the first peak is centered at *x*=40.7. This phenomenon is caused by the wall moving rightward in the quantum well. The non-periodic revival pattern is a consequence of energy dissipation due to the moving wall.

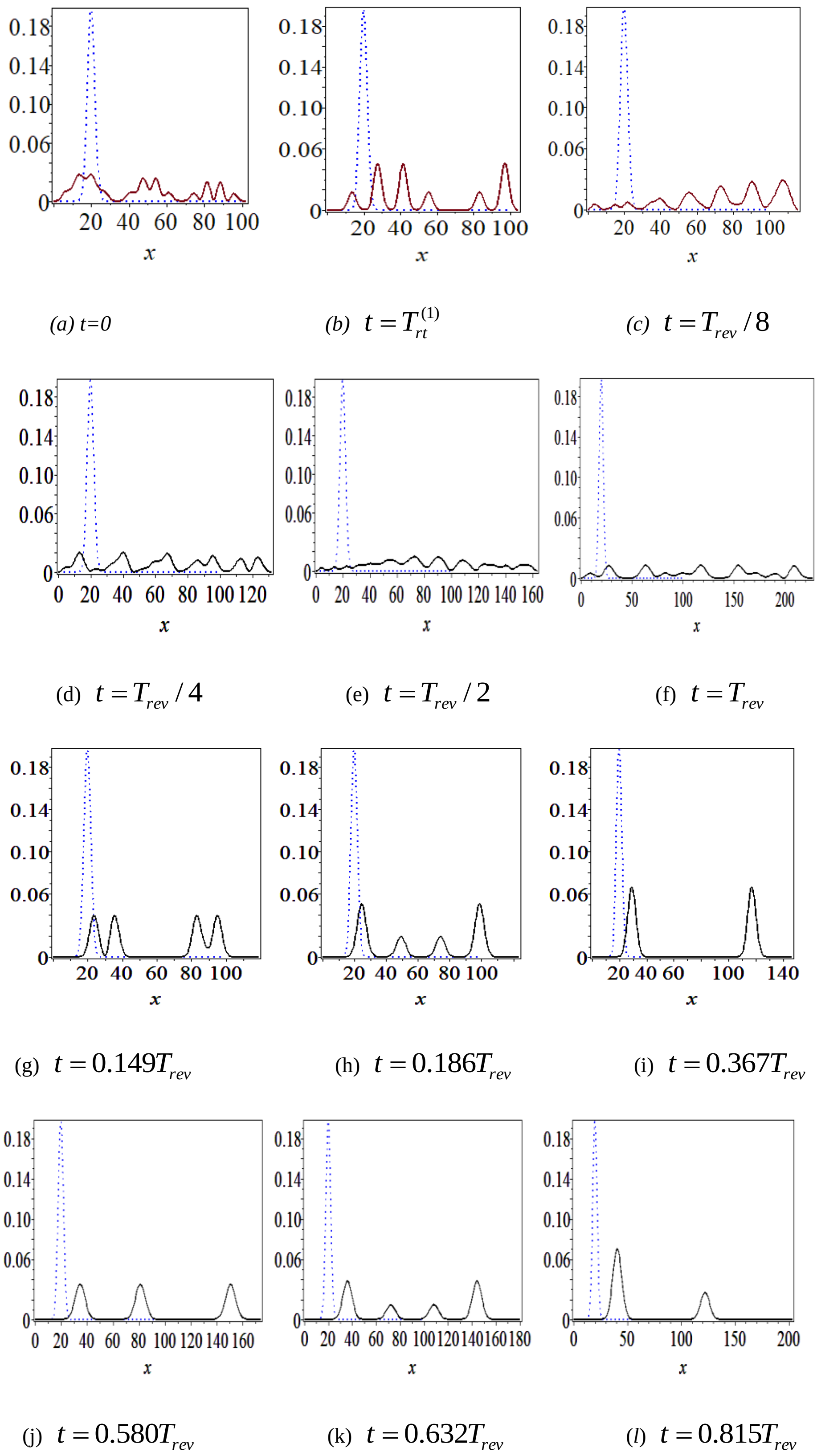
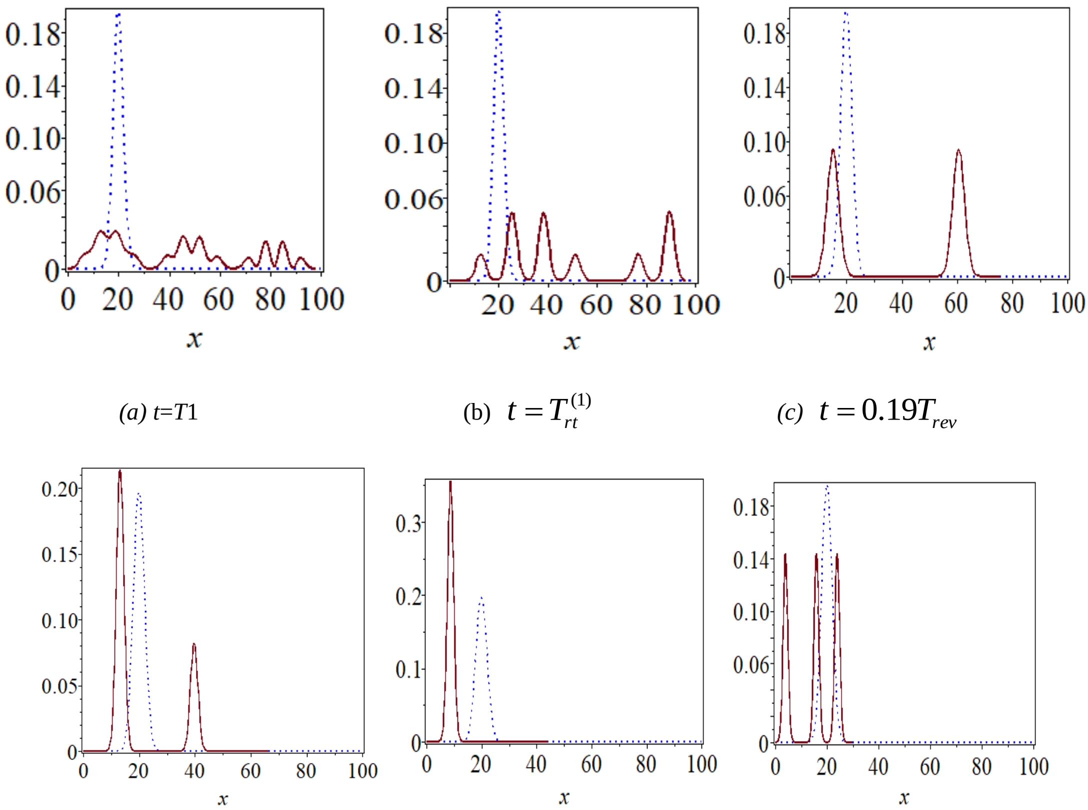


Fig. 4. (color online) The probability density distribution $|\psi(x,t)|^2$ *versus x in the quantum well with one wall moving fast. The moving speed of the wall is v=0.01.The times are given in each plot*

As a comparison, in Fig. 5, we calculate the probability density distribution with one wall moving leftward at a constant speed *v*=-0.01. Under this condition, the time when the particle first hits the moving wall is $t = T_1 = \frac{l_0}{k - v} = 207.80$. The first round-trip period of the particle in the quantum well is $t = T_{rt}^{(1)} = \frac{2l_0}{k_0 - 2v} = 407.13$. Fig. 5 (a) is the probability density distribution at $t = T_1$. The wave packet is collapsed and expanded in the whole quantum well. At the first round-trip period, six distinct peaks appear in the probability density distribution. Compared with the case that the wall moves rightward, the whole wave packet moves towards the left. As time goes on, partial revival of the wave packet takes place, but the first peak in the probability density distribution moves leftward relative to the initial wave packet, which is caused by the wall moving leftward in the quantum well. In addition, with the increase of the time, the probability density distribution region in the quantum well becomes decreased. For example, in Fig. 5 (f), $t = 0.55T_{rev} = 7000$, the first peak moves towards the left wall and the probability density distribution is limited in a small region, 0<x<30.The initial wave packet can not be reconstructed completely.



(a) t=T1 (b) $t = T_{rt}^{(1)}$ *(c)* $t = 0.19T_{rev}$

(d) $t = 0.265T_{rev}$ *(e)* $t = 0.44T_{rev}$ *(f)* $t = 0.55T_{rev}$

Fig. 5. (color online) The probability density distribution $|\psi(x,t)|^2$ *versus x in the quantum well with one wall moving leftward. The moving speed of the wall is v=-0.01.The times are given in each plot*

Finally, we calculate the absolute square of the autocorrelation function of this system in a revival period: $|A(t)|^2 = |\sum_n |c_n|^2 \exp[i\int_0^t E_n d\tau]|^2$. $|A(t)|^2$ is also called survival probability, which is varied between 0 and 1. If $|A(t)|^2$=0, which suggests that the state $\psi(x,t)$ has a shape that is distinct from the initial state $\psi(x,0)$.

However, when the final state $\psi(x,t)$ exactly regains the initial state $\psi(x,0)$, $|A(t)|^2=1$.

Firstly, we suppose the initial momentum of the Gaussian wave packet $P_0=0.0$.

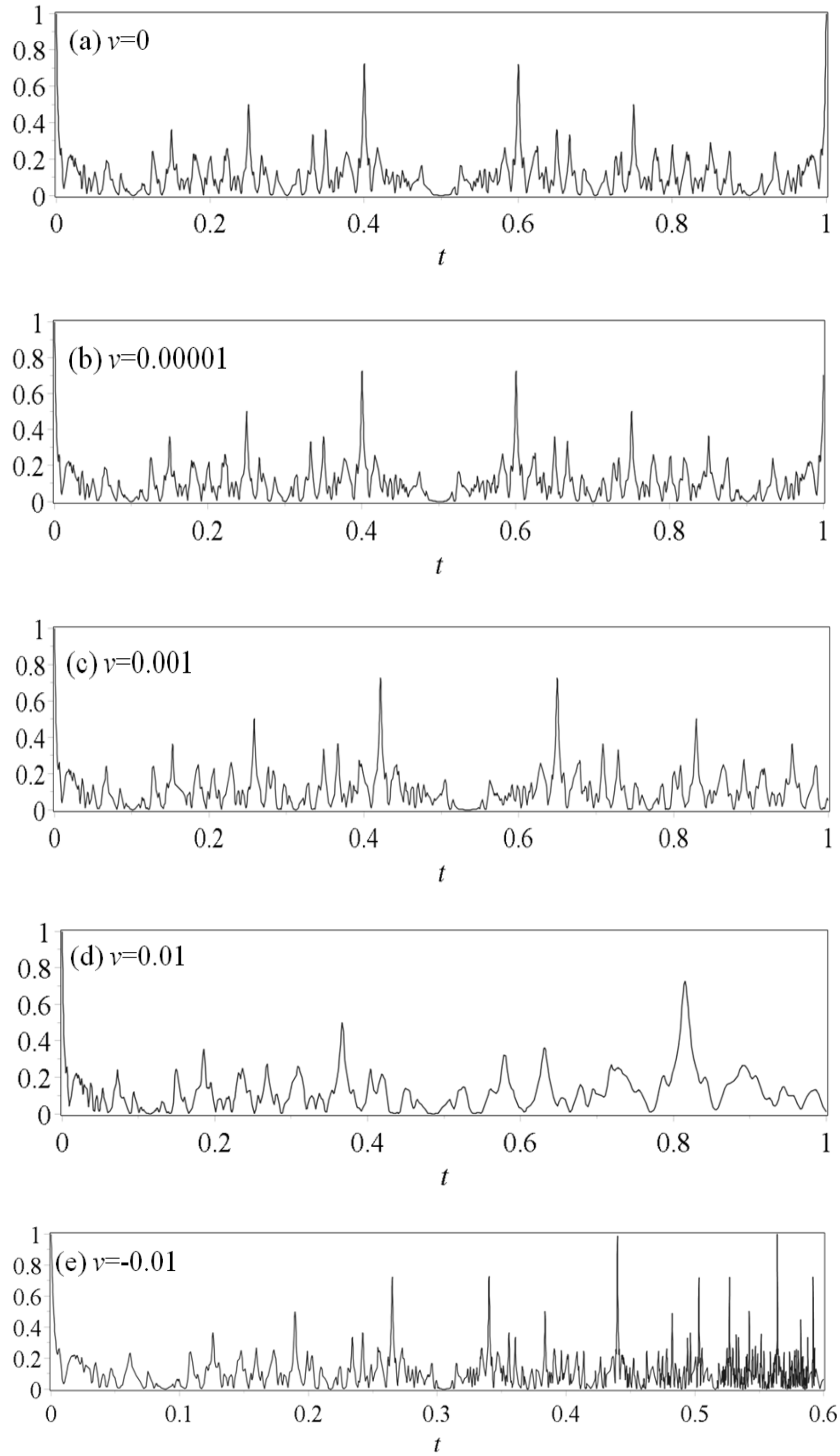


Fig. 6. Variation of the absolute square of the autocorrelation function of this system $|A(t)|^2$ *in a revival period. The moving speed of the wall is given in each subplot. Suppose the initial momentum of the Gaussian wave packet* $P_0=0.0$

Fig. 6 (a) shows $|A(t)|^2$ in the quantum well with two static walls. It is found the survival probability is symmetric relative to the middle of well. At one revival period, $t=T_{rev}$, $|A(t)|^2=1$, an exact wave packet revival occurs. Fig. 7 (b) shows $|A(t)|^2$ in the quantum well with one wall moves very slowly along the +x axis, the speed of the moving wall is v=0.00001. Under this condition, the effect of the moving wall on the wave packet dynamics in the quantum well is very small. As $t<T_{rev}$, the difference of the above two plots can be neglected. The only difference takes place at $t=T_{rev}$. In Fig. 6 (b), $|A(t)|^2<1$, which suggests at one revival period, only partial revival appears in the wave packet when one wall in the quantum well is moving rightward. With the increase of the moving speed of the wall, its effect on $|A(t)|^2$ becomes apparent. Only partial revival appears in the autocorrelation function and the full revival phenomenon in the wave packet does not appear. In addition, the interval between the adjacent revival peaks in the autocorrelation function increases with the moving speed. As we can see from Fig. 6 (c-d). Fig. 6 (e) shows $|A(t)|^2$ in the quantum well with one wall moves leftward, the speed of the moving wall is *v*=-0.01. In this plot, we find at $t=0.44T_{rev}$, $|A(t)|^2=1$. But from the probability density curve shown in Fig. 5 (e), we find at this time, although only one peak appears in the probability density, but the position of the peak is deviated from the initial wave packet. The full revival of the wave packet cannot be observed, which can be considered as a moving wall effect on the wave packet dynamics.

Next, we fix the speed of the moving wall, *v*=0.01, then we show how the autocorrelation function of this system vary with the initial momentum in the Gaussian wave packet.

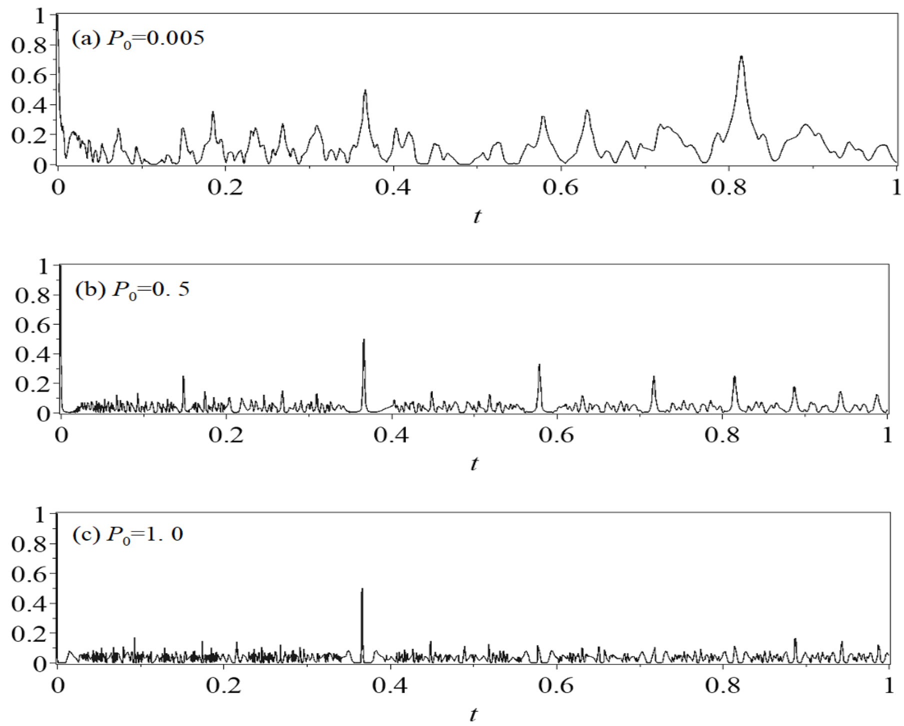


Fig. 7. Dependence of the absolute square of the autocorrelation function of this system $|A(t)|^2$ in a revival period on the initial momentum of the Gaussian wave packet. The moving speed of the wall is v=0.01. The initial momentum of the Gaussian wave packet is given in each plot

The results are shown in Fig. 7. Fig. 7 (a) shows the autocorrelation function with the initial momentum $P_0 < v$, P_0=0.005. Its influence on the autocorrelation function is very small compared to the zero momentum case shown in Fig. 6 (d). The autocorrelation function decreases somewhat more rapidly form its initial values than in the P_0=0 case as the initial momentum $P_0 > v$, P_0=0.5(Fig. 7 (b)). For still larger value of momentum, such as P_0=1.0 (Fig. 7 (c)), the particle is moving away from its initial position very fast. It can be noted that the interval between the adjacent peak in the autocorrelation function decreases. In addition, only one recurrence peak appears obviously in the autocorrelation function. This figure suggests that we can control the evolution of the wave packet in the quantum well with one moving wall by changing the initial momentum of the Gaussian wave packet.

4. Conclusions

In summary, we have investigated the wave packet dynamics for a system with one moving wall in the quantum well. Revival and partial revival of the wave packet in this system are quite different from the case in the static quantum well, which depends on the moving speed of the wall sensitively. The fractional revival phenomenon of the wave packet is shown to exist when the wall is moving very slowly, but the full revival of the wave packet is disappeared forever. As we increase the speed of the moving wall, a peculiar revival pattern occurs, there are no partial revivals at half, a quarter…… of the revival time. Instead, partial revivals are observed at some special decimal fractions of the revival time. When the wall is moving left, the full revival is predicted through the auto-correlation function curve, but a plot of the probability density curve shows non-identical peak where full revival is expected. It turns out that the position of the revival peak deviates from the initial wave packet, which depends on the motion direction of the moving wall. These non-periodic revival patterns observed in this system can be considered as a consequence of the moving wall effect.

It is interesting to study the wave packet dynamics in the presence of a moving wall since there is much work being carried out related to the moving boundary problem. Our work provides a better understanding of the temporal evolution of wave packets in the quantum well with one moving wall by comparing with a familiar system. In this work, we only deal with the simplest case of one wall moving with constant velocity. For more general surface motions, such as accelerating wall motion or oscillatory wall motion, the method used in this work is still suitable. In our future studies, we will study the case in which the wall moves accelerating or oscillating with time and compare our results with those given by Ref. [37-38]. We hope that our work can provide some references for the future experimental study of the wave packet dynamics for a system with a moving boundary.

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*Corresponding author: lduwdh@163.com